A Simple Algorithm for Constructing Perfect Monolinear *Sona* Tree Drawings, and its Application to Visual Art Education

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Abstract: - The traditional sona drawing art of Angola is compared from the structural point of view to similar geometric arts found in other parts of the world. A simple algorithm is proposed for constructing a sub-class of sona drawings, called *perfect* monolinear sona drawings, on a given set of points in the plane, that exhibit the topological structure of a tree. The application of the tree sona drawings obtained with this algorithm to the visual arts and Discipline Based Art Education is illustrated.
Key-Words: - Sona drawings, monolinear drawings, minimum spanning trees, dual tree, algorithms, kolam art, sand-drawings, Celtic knots, visual arts, art education

1 Introduction

Many cultures all over the world have traditional visual art and design practices that unite them with a common thread: the use of geometric curves with certain geometric properties [19]. Such practices include *sona* drawings from Angola in Africa [9, 10], *kolam* drawings from South India [18, 33], Celtic knots [22, 26], Sand drawings from the South Pacific Islands [34], and Chinese knots [19].

A sona drawing consists of a single curve that starts at one point, meanders about a group of points previously drawn, and ends at the same place it started. Sometimes more than one curve is drawn, but the most challenging, and thus prized, drawings consist of one curve, and are called *monolinear*. Another desirable property of these drawings is that every region must contain exactly one point. Such a sona drawing will be called *perfect*.

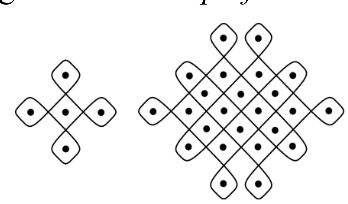

Fig. 1: Two symmetric sona drawings.

An additional property that sometimes is sought is that the sona drawing should exhibit some form of symmetry. Two such perfect sona drawings are pictured in Figure 1. They exhibit mirror symmetry with respect to both the vertical and horizontal lines through the centers of the figures. It follows that this art has a strong geometric flavor, and that it uses a variety of rules to produce the drawings. In Angola many such drawings belong to an old artistic oral tradition that accompanies story telling, making reference to proverbs, fables, games, riddles, animals, and social rituals such as circumcision. [9].

Drawings similar to sona drawings are used in other cultures besides Angola. For example, in the ancient Tamil tradition of South India they are called Kolam [29, 33]. Similar drawings are also found throughout the northern and central islands of Vanuatu in the Pacific, where they are called *Malekula* sand drawings because they are tylically drawn in the sand, and are part of the cultural tradition of the *Malekula* people [8, 34].

In this paper first the traditional sona drawing art of Angola is compared from the structural point of view to similar geometric arts found in other parts of the world. A simple algorithm is then proposed for constructing a sub-class of sona drawings, called *perfect* monolinear sona drawings, on a given set of points in the plane, that have the topological structure of a tree (hereafter called *tree sona*). Finally, the application of the tree sona drawings obtained with this algorithm, to the visual arts and Discipline Based Art Education is also illustrated.

The type of research considered here falls in the general area of *interdisciplinary academic* research connecting *computer science*, *studio work* and *visual art education* components [13]. For one, it provides a relatively simple method of transforming an old traditional art form into a contemporary setting. Secondly, the geometric and symmetric constraints imposed by the rules for producing the final art works challenge visual problem-solving abilities, expand visual thinking, and inspire creativity [1, 5, 7, 27]. Thirdly, the topic creates an interactive bridge between art and technology [25, 28]. Studying sona drawings offer an opportunity to explore this interaction. In addition, the structural approach considered here can be used to categorize the sona drawings used in the Angolan tradition, in much the same way that Dorothy Washburn used categorization methods, using symmetry properties, to classify the geometric patterns in Bakuba raffia cloth found in Central Africa [32]. Finally, the tools developed here can help to make historical research more quantitative [3, 4, 15, 16, 21].

2 Review of the Literature

It is difficult to find other research similar to that described here, that uses sona drawings. Some research may be found on related geometric arts, such as Celtic knots or Roman mosaic floors, but these works are mostly limited to documenting the types of patterns used, and where they are found, generally ignoring the exploration of methods for creating new similar patterns. Some work has also been done in trying to describe the influence of one culture over another in pattern design. But no work exists that focuses on using such a traditional art to create new directions and apply them to studio art and visual art education. What follows is thus a brief review of some work related to this project.

That sometimes it is interesting to turn our attention away from our own artistic products of, what some call, an advanced civilization, to examine the handicrafts of what some authors have referred to as "primitive societies," has been documented by some researchers [14]. Joyce analyzed a large collection of embroideries of the *Bushongo* people of Central Africa, found in the British Museum. She concluded that "*it is salutary to recognize that, however much the resources of civilization may have contributed towards the improvement of technical processes in manufacture, they have sometimes been unable to make any marked addition to the artistic value of the finished product."*

More closely related to sona drawings are the sand drawings of the South Pacific Islands [34]. In many of the islands of Vanuatu in the South Pacific, the people engage in a cultural practice of creating drawings by tracing geometric diagrams in sand, dust, or ashes. The drawings have to be made without lifting the finger and can be quite difficult to accomplish. These drawings are much more than decorations or simple pictures, and could well be considered as a form of language writing. They are often accompanied by story-telling, and are used by parents to teach cultural values to their children.

The ancient floor mosaics of the Romans used interlace art related to sona drawings. In the period of 1965 to 1990 spectacular Roman floor mosaics were discovered throughout the island of Cyprus, generating great excitement [23]. These decorative patterns consist of interlaced rope designs that are somewhat similar to sona drawings. However, they are different from many Celtic knots and Sona drawings in that they consist of many cyclic curves rather than just one.

Another topic that relates to sona art is the topic of decorating a page or an initial letter in an ancient manuscript, with gold and silver, or colored designs [12]. This is technically called *'illumination of codices.'* The manuscripts discussed in this paper are Mozarabic, that is, from the Christian inhabitants of Spain that lived under the Muslim

Moorish kings. In particular Guilmain deals with manuscripts dating from the end of the 9th century onwards, and restricted to the region in Spain known as Leon-Castile. The unusual character of these manuscripts has been of interest to art historians for a long time. The author documented whether the interlace patterns used were composed of one, two, or more curves. He also compared the *complexity* of the patterns, an interesting topic in itself. Finally, in order to find connections between seemingly quite different patterns, he searched for methods of constructing them from similar underlying grids. This paper analyzes some geometric structural properties, and provides some rules for obtaining and comparing different knot patterns. However, no precise algorithm is given that can be programmed to generate the patterns.

Art found on the murals of the homes of the Pedi peoples of South Africa is also related to sona drawings [31]. Vogel analyzes the structure of the patterns, their effects on the architecture and perception by the persons entering the dwellings, as well as how these structures reflect the society's cultural values. The author classified the geometric patterns found into 33 categories depending on such features as straight lines, curved lines, chevrons, diagonals, ripple-lines, crescents, double crescents, diamonds with a dot in the center, etc.

Celtic knots are also closely related to sona drawings. The two main distinctions between them are that in Celtic knots the curve has a 3-dimensional aspect because the strands pass under and over each other in an alternating pattern [6, 17]. This is usually not the case with sona drawings, although there are some notable exceptions. For example a common sona drawing used in several different contexts is the pattern on eight points shown in Figure 2.

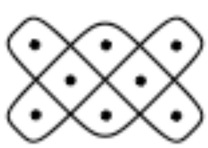

Fig. 2: A perfect sona drawing on eight points.

However, this pattern is often represented with the over-under 3-dimensional knot aspect prevalent in Celtic knots, in the form illustrated in Figure 3.

Fig. 3: A drawing with an over-under aspect.

In addition, the points used in sona to guide the drawings are absent in celtic knots. There has also been some work on generating Celtic knots by computer [26]. The author in [26] analyzed the algorithms developed by others in constructing Celtic knots by hand, found in the book: *Celtic Knotwork* by Ian Bain (Constable, London, 1986). These methods are based on using grid cells, reference skeletons, and plait-breaking rules to create knot compositions. This thesis contains a lot of history about Celtic knots, and it gives several examples of contemporary artists' work that are inspired by Celtic knot designs.

In the modern Western context, Sona drawings may be considered as a sort of minimalist art, or low complexity art [30]. For example, a modern art form in some sense similar to sona drawings was developed by Robert E. Mueller [24]. The research question discussed in Mueller’s article concerns what he calls *'schemas'*. Schemas are black and white drawings of dots and lines with black ink on white paper. The fundamental question Mueller discusses is: are schemas a successful minimal art form? The artist mainly describes his own evolution in this direction starting at about 1960.

Much more closely related to sona drawings are Kolam drawings from south India [18, 19, 29, 33]. These papers are the most relevant to our algorithm proposed here, and deal with the mathematical analysis of the patterns. They also describe some rather involved procedures for generating new sona patterns. Robinson [29] uses the theory of formal languages to define some “*pasting*” rules, to paste together simple sona to create more complex sona. Yanagisawa and Nagata [33] describe an algorithm for generating sona patterns that comes closest to our algorithm but is much more involved. First they define 16 simple constituent units defined on 5 points that are found in sona drawings. Then they define a set of 5 rules for combining the 16 elements to create sona drawings. One severe limitation of their procedure is that it works only for points that are drawn on a regular grid. The algorithm presented in the next section is not only extremely simple, but works for any set of points in the plane, no matter how irregularly they are distributed.

3 A Simple Algorithm for Tree Sona

A sona drawing may be considered as a sort of *dual* to a graph defined on the original set of points that acts as a guide for the creation of the sona drawing. For example, in Figure 1 (left) each closed region contains a point. If we connect each pair of points whose regions touch each other at an intersection point, by a line segment, we obtain the dual graph, which in this case is a tree. This sona drawing is thus an example of a *tree sona.* Apart from this simple example on 5 points, tree sona are notoriously absent in the sona tradition of Angola [9, 10]. This is surprising given that a tree is such a fundamental structure. This is one motivation for studying tree sona.

With the algorithm proposed here we can easily make new, interesting sona drawings that have not been used in the traditional sona drawing art or related arts. For example, consider the two tree sona shown in Figures 4 and 5. There is a simple local algorithm (easily discovered by the reader) that one can follow, given the points, to create the mono-linear drawing of Figure 4. However, it will work only for this kind of tree on this pattern of points. Furthermore, this drawing is not symmetric. On the other hand, the symmetric mono-linear drawing in Figure 5 appears to have a very difficult rule for tracing the curve, as anyone may verify by trying it. Nevertheless, the algorithm proposed here and described in the following uses only two very simple local rules to create such complex sona drawings on any set of points no matter how irregularly they are distributed. The algorithm will be described by illustrating its application to an example set of points.

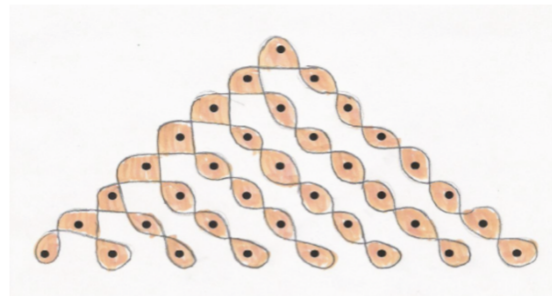
Fig. 4: A non-symmetric sona tree.

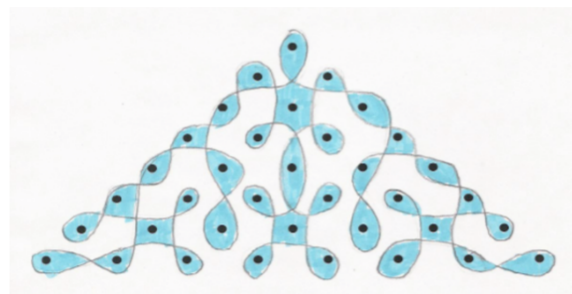
Fig. 5: A symmetric sona tree.

Therefore assume we have a set of points in arbitrary position, such as that illustrated in Figure 6. Before drawing the sona curve, the final kind of tree structure desired must be chosen. For this it is sufficient to first construct a spanning tree of the set of points that will be used as a guide in the final construction of the sona drawing. For the algorithm to work, any spanning tree will do. However, if the application is visual arts, then a "nice" tree should be selected. A *minimum spanning* tree usually gives nice results, but the artist may prefer certain structures such as those seen in Figures 4 and 5, depending on the application. In Figure 6 such an arbitrarily selected tree is shown in dotted lines. For definiteness, the algorithm for drawing the tree starts on any point that is a leaf in the tree such as point ***s*** in Figure 6. The procedure followed resembles a kind of depth-first search of the tree starting at point ***s***. The sona curve meanders from counter-clockwise to clock-wise contour tracing, crossing the edges of the spanning tree as it proceeds until a vertex of degree higher than two is encountered, such as vertex ***a***. When vertex ***a*** is encountered along edge ***xa*** one of the adjacent edges ***ay*** or ***az*** must be visited next. The one selected is the one being pointed to by the sona curve, in this case ***ay***. This procedure is followed until the point ***s*** is encountered again. The final sona drawing is shown in Figure 7. Note that the number of edges in the sona regions of each point equals the degree of the vertex. The region of point ***a*** has 6 sides, that of point ***b*** has 4 sides, that of point ***c*** has 3 sides, the leafs have one side and the rest have 2 sides.

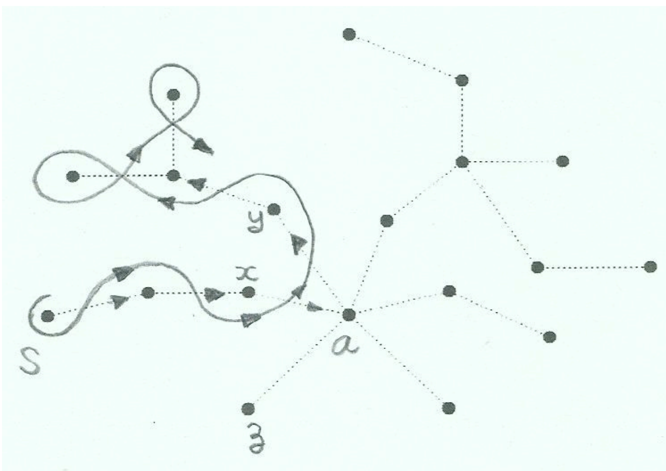

Fig. 6: The algorithm starts on the leaf labeled ***s***.

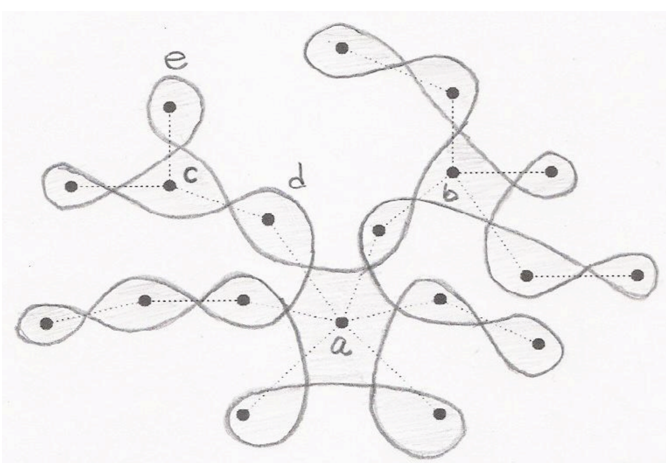

Fig. 7: The completed sona drawing.

4 Creating New Art Using Sona

In this section we briefly illustrate how we can use sona drawings to create new art in a modern setting. Clearly, new art can be created by deleting, adding, or changing some of the components of sona drawings. This involves a personal journey of exploration and personal symbolic meanings that artists may want to add to their art works. Some examples of the kinds of changes that an be made include the following:

The nature of the curve: It is possible to draw the curves in many different ways. The most obvious methods are either very precisely using an underlying grid, or with more free-hand variability, avoiding sharp curves. The curves could be drawn close to the points, or keeping their distance from them. The curve could also be drawn with varying thickness, colors, and textures.

The nature of the dots: It is possible to either delete the dots altogether after drawing the curve or leave them there. The points can also be made of different sizes, colors, textures, and shapes.

The nature of the background: The curve divides the space into the interior parts that hug the points, and into the exterior background outside the regions that contain the points. These spaces may also be colored with different colors, and textures.

The overall structure of the design: Once the steps outlined above have been finished on a chosen set of points, other such sets may be completed and connected together to form additional higher level structures and symmetries.

Sona drawings and their relatives fit well in several philosophies of art education including *Discipline Based Art Education* (DBAE), with the added components of cross-cultural elements, art history, and aesthetics [11]. In particular, sona drawings are ideal for teaching a problem-solving approach to art that is interdisciplinary [1, 7]. Also relevant is that, because of their algorithmic construction, sona drawings can be used in the study of art as *process* rather than *product*.

To close this section the process outlined above is illustrated with a couple of artworks by the first author. The first artwork titled "*QuoVadis*" painted by Yang Liu in 2008 shown in Figure 8 consists of a single sona curve that varies in thickness and texture as it evolves. The original points have been deleted. The second artwork titled "Pottery Study No. 1" also painted by Yang Liu in 2008 shown in

Figure 9 consists of an arrangement of 8 sona trees with their points deleted and some with their interiors painted white.

Fig. 8: "*Quo Vadis*" by Yang Liu (2008).

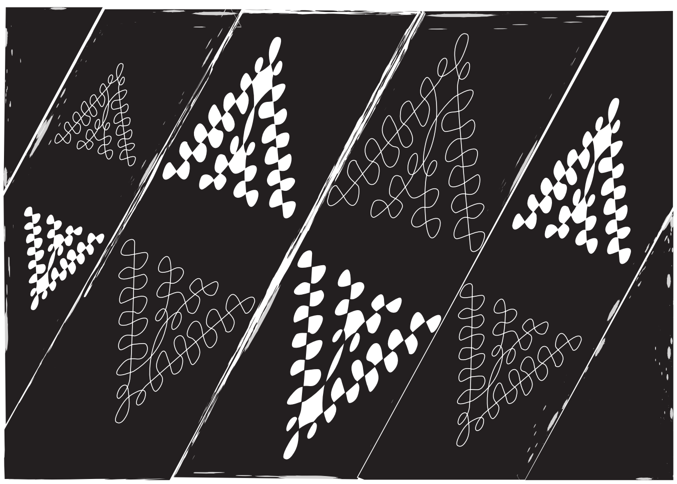

Fig. 9: "*Pottery Study No. 1*" by Yang Liu (2008).

5 Conclusion

A very simple algorithm for constructing perfect sona tree drawings from a set of points was described, and it was shown how the algorithm can be used as a tool for artists to create new art.

In addition to the computer science and art aspects of this project, there are also social implications that arise from the fact that a non-Western traditional cultural art is being analyzed, extended, and modified using modern Western concepts and ideas. The result is that both cultures are transformed by this artistic process, thus providing an intercultural connecting bridge. The kind of work involved in creating such artwork also provides a suitable method to teach cross-cultural and interdisciplinary aspects of art. It encourages collaboration between the arts and the sciences [28], particularly in the aspect of searching and generating the different kinds of objects (trees in this case) that have certain geometrical properties relevant to art, such as symmetry. It is very well suited also to a problem-solving approach to art education. It has been reported in the literature that experiences in problem solving accelerate the students' personal growth in the arts [1]. As such it helps to resolve some of the conflicts that some art educators perceive exist between art and science [20]. The work described here also fits in well with the *expansive* role of visual arts curricula, which aims to incorporate a variety of intelligences and modes of thinking [2].

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