Computational Geometry

Assignment 5

Date Due: Tuesday, December 3, 2002
Time Due: In Class
All questions are of equal value. Your proofs should be precise, concise and clear. Assume that all polygon vertices are given in counter-clockwise order. If you need to assume general position for a set of points (that is no three points are collinear), clearly state so and briefly discuss if the assumption is important. When describing an algorithm, do so in words rather than code.

1. A tripod is defined as a point p with three rays emanating from p such that the angle between two consecutive rays is at most π. Notice that a tripod partitions the plane into three regions (i.e. cones). Given a set S of n points in the plane (in general position), is it possible to find a placement of a tripod such that each region contains at most $\lceil n/3 \rceil$ of the points? If it is possible, then prove that a valid placement always exists, design an algorithm for doing this, prove the running time and correctness of the algorithm and give a lower bound for the problem. If it is not possible, then provide a set of n points and a tripod T and prove that there is no placement of T which has the required properties.

2. Notice that in the standard construction of a 2-dimensional k-d tree, between each level of the tree, we alternate between horizontal and vertical splitters. If you look at the algorithm for constructing a k-d tree (on page 100 of the second edition of the class text), you see that this is decided at step 3. When the depth is even, a vertical splitter is used and when the depth is odd, a horizontal splitter is used. Suppose that I modify this algorithm in the following way: In step 3, instead of verifying the parity of the depth, I flip a coin and use a vertical splitter when I get heads and a horizontal splitter when I get tails. Determine what is the expected and worst case number of intersections between a query rectangle and a k-d tree built in this way.

3. Prove that the minimum spanning tree of a set of n points is a subgraph of the Delaunay triangulation of that point set.

4. Given n points in the plane $p_1, \ldots, p_n$, the hypermedian slope is computed as follows: for each point p_i, compute the slope of the line that connects it to all $n-1$ other points, and let m_i be the median of those $n-1$ slopes. The hypermedian slope m^* is the median of the values $m_1, \ldots, m_n$.

   (a) Give a description of the points, the m_i's and the hypermedian slope in the dual.

   (b) Given a vertical line $x = t$ in the dual, give an efficient algorithm to decide if $m^* \leq t$.

   (c) Give an efficient algorithm to compute m^*.