Computational Geometry

Assignment 4

Date Due: Thursday, November 7, 2002
Time Due: In Class

1. Given a set P of n points in general position, define a graph G as follows. The vertex set is P. Two vertices, a and b, are joined by an edge provided there exists an axis parallel square S with a and b on the boundary and no other point of P in the interior of S. Prove or disprove that G is a triangulation.

2. Given a set of n line segments in the plane, determine whether or not it is possible to compute a triangulation of the endpoints of the line segments such that each of the segments is an edge of the triangulation. If it is possible, then provide a proof that it is possible, provide an algorithm to compute such a triangulation and give an upper bound on the running time of the algorithm. If it is not possible, provide a counter-example, i.e. a set of n segments which are impossible to triangulate.

3. In class, we saw how voronoi diagrams can be used to solve the following problem efficiently: Given a set of n sites and a query point, report the site *closest* to the query point in $O(\log n)$ time. What if we wanted to report the *furthest* point instead. Show how to preprocess n sites into a data structure so that you can quickly report the point furthest away to a query point.

4. We saw in class that the incremental Delaunay triangulation construction on n points performs an expected $O(n)$ number of flips over the entire construction if the ordering in which the points are inserted is a random permutation of the input points.

   (a) Find a set of points and a bad ordering that will make the algorithm perform as many flips as possible. Is your example the worst?

   (b) Given any set of points, is it possible to find a bad ordering, or are there sets of points for which every ordering will be good?

5. The Voronoi diagram of a set of points is drawn with straight line segments and rays. How about the Voronoi diagram of

   (a) A set of lines?

   (b) A set of lines and points?

   (c) A set of line segments?

   (d) A set of line segments forming a convex polygon?

   (e) A set of line segments forming a simple polygon?

   (f) A set of circles (non intersecting and no circle contains another)?

   For each of these sets of objects, describe the shape of the boundary pieces that form the Voronoi diagram. (The distance between a point and another object is the smallest distance between the point and any point of the object).