Computational Geometry

Assignment 3

Date Due: Thursday, October 17, 2002
Time Due: In Class

1. How quickly can you sort the x coordinates of the vertices of a given polygon P? Give the best lower bounds and algorithms you can find, when:
   (a) P is a convex polygon,
   (b) P is a simple polygon,
   (c) P is a simple polygon and the convex hull of P is provided.
   (d) P is a simple polygon and a triangulation of P is provided.
   (e) P is a y-monotone polygon
   (f) P is an x-monotone polygon
   (g) P is a star-shaped polygon (i.e. there exists a point q that can see every vertex of P)
2. Let Q_n be a simple n-gon and suppose you are given a set of k non-crossing diagonals that partition its interior into the union of quadrilaterals with disjoint interiors.
   (a) Can n be odd?
   (b) What is k? How many quadrilaterals will there be?
   (c) How many guards are sufficient to guard Q_n? Does this violate the art gallery theorem?
3. Given n axis-parallel rectangles in the plane,
   (a) What is the maximum number of pairs of rectangles that could intersect?
   (b) Give an $O(n \log n)$ algorithm to find the rectangle that intersects the largest number of other rectangles. Is it optimal?
4. Let $T = (v_0, v_1, v_2)$ be a triangle. Let P be a set of n points in T. Build a triangulation on $P \cup \{v_0, v_1, v_2\}$ in the following incremental fashion.
   (a) Select at random a point p of P. Remove p from P.
   (b) Find the triangle $\triangle(a, b, c)$ of T that contains p.
   (c) Update T by adding the edges $[pa], [pb], [pc]$.

   In order to perform the second step efficiently, we can build a corresponding search tree to speed up the point location. Let the root of the search tree represent the initial triangulation. Notice that each update to the triangulation has a corresponding update in the tree where we add three nodes (the three new triangles) to a leaf (the triangle that contained the point). To locate which triangle contains the update point, simply search in the tree until a leaf (which represents a triangle in the current triangulation) is reached. After all n points have been added, the search tree is a point location structure for the final triangulation.

   Given a query point, what is the expected and worst case time to perform a point location? What is the expected and worst case time to perform the entire construction? Lower bounds for constructing any triangulation of $P \cup \{v_0, v_1, v_2\}$?

5. Given a set of n axis parallel boxes in R^3, preprocess these boxes to answer the following query quickly. Given a line L, parallel to the X axis, report all boxes that intersect L. What is the size of your data structure? How long does it take to build your data structure? What is the query time?

6. Let $s_1, s_2, \ldots, s_n$ be a set of n squares in the plane.

   (a) Design a data structure to quickly determine if a query point $p \in \cup_{i=1}^{n} s_i$. What is the size of the structure, how long does it take to build it and what is the query time?

   (b) Design a data structure to quickly determine if a query point $p \in \cap_{i=1}^{n} s_i$. What is the size of the structure, how long does it take to build it and what is the query time?