

## COMP-MATH-22 Discrete Mathematics

### Practice Problem 1:

#### The Collapsing Compass

In proposition II of *Book I* of *The Elements*, Euclid proposed an algorithm (using the *straight-edge-and-collapsing-compass* computer) to *transfer* (or displace) a given line segment  $[a,b]$  to a new location so that one end point, say  $a$ , lies on a pre-specified point  $c$  in the plane. The transferred segment obtained by executing Euclid's algorithm is not necessarily *in any particularly specified direction with respect* to the original segment.

(a) Design an algorithm that uses the same computer (the *straight-edge-and-collapsing-compass*) and necessarily yields a displacement so that not only  $a$  lies on a pre-specified point  $c$ , but such that it satisfies the additional requirement that the new position of the segment be *parallel* to the old position. The Lemoine simplicity of an algorithm is defined as the number of circles plus the number of lines drawn. What is the Lemoine simplicity of your algorithm? The lower the Lemoine simplicity of your algorithm, the better.

(b) Prove the correctness of your algorithm.