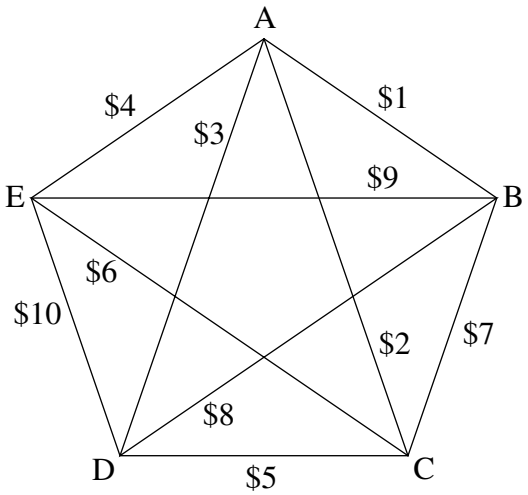
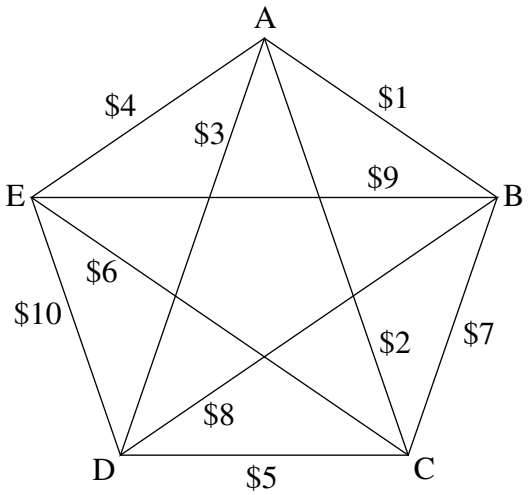

A
$4
$1
$3
$9
E
B
$6
$10
$2
$7
$8
D
$5
C

Data Structures and Algorithms CS 308-251 A - 1999

Problem Assignment #6 ***(due December 2, 1999)***

1. Greedy Algorithms and Least Cost Hamiltonians *(2 points)*

Consider the Traveling Salesman Problem for the network shown below. The Greedy algorithm for computing a least-cost Hamiltonian cycle starting at some vertex X always chooses the least costly edge to travel to the next city. Show for this network that no matter which vertex is used as a starting vertex, the Greedy algorithm will never find the Hamiltonian cycle of least cost.



2. Hamiltonian Cycles in Dense Graphs *(4 points)*

Ore's Theorem states that: if a connected graph $G=(V, E)$ with n vertices (n greater than or equal to 3), is such that for each pair of non-adjacent vertices v and w we have that

$$d(v) + d(w) \geq n$$

then G contains a *Hamiltonian cycle*. In class we saw a proof of this theorem by *backwards induction*. Convert this proof to an algorithm for computing a Hamiltonian cycle in such graphs in $O(n^2)$ worst-case time. Describe the algorithm, the data structures used, and the complexity analysis in detail.

3. Convex Hulls of Points *(4 points)*

The input is a set S of n points in the plane in general position, i.e., no three points lie on a line. Design an $O(n)$ worst-case time algorithm to determine if a new point X lies inside the convex hull of S. Prove the correctness of your algorithm. Prove the complexity of your algorithm.