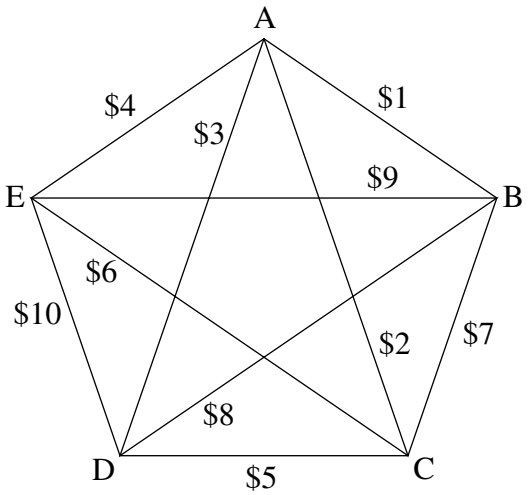
A
$4
$1
$3
$9
E
B
$6
$10
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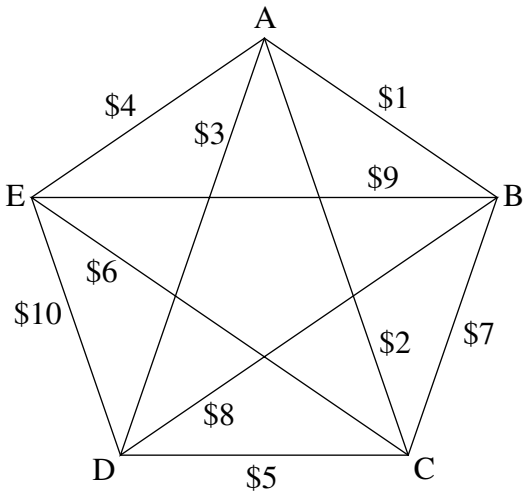
$7
$8
D
$5
C

Data Structures and Algorithms CS 308-251 A - 2000

Problem Assignment #6 *(due November 30, 2000)*

1. Greedy Algorithms and Least Cost Hamiltonians

Consider the Traveling Salesman Problem for the network shown below. The Greedy algorithm for computing a least-cost Hamiltonian cycle starting at some vertex X always chooses the least costly edge to travel to the next city. Show for this network that no matter which vertex is used as a starting vertex, the Greedy algorithm will never find the Hamiltonian cycle of least cost.



2. Hamiltonian Cycles in Dense Graphs

Ore's Theorem states that: if a connected graph $G=(V, E)$ with n vertices (n greater than or equal to 3), is such that for each pair of non-adjacent vertices v and w we have that

$$d(v) + d(w) \geq n$$

then G contains a *Hamiltonian cycle*. In class we saw a proof of this theorem by *backwards induction*. Convert this proof to an algorithm for computing a Hamiltonian cycle in such graphs in $O(n^2)$ worst-case time. Describe the algorithm, the data structures used, and the complexity analysis in detail.

3. Prim's Minimum Spanning Tree Algorithm for Graphs

Suppose that the graph $G=(V, E)$ with n vertices is represented as an adjacency matrix. Give a simple implementation of Prim's algorithm for this case that runs in $O(n^2)$ worst-case time.