# ACEPT Co (Integrated Inventory-Transportation Optimization)

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# Problem Statement

- ACEPT Co. ,a manufacturing company, needs to deliver its product to its customers.
- It would like to optimize its operations costs. Specifically those associated with storing and then delivering its product to the customers.

# Problem Statement

- The customers each require a certain amount of product each day, the daily demand.
- In addition the customers each have a certain amount of storage/inventory space. This inventory space is limited and the company cannot deliver more product than the maximum capacity of each customer.

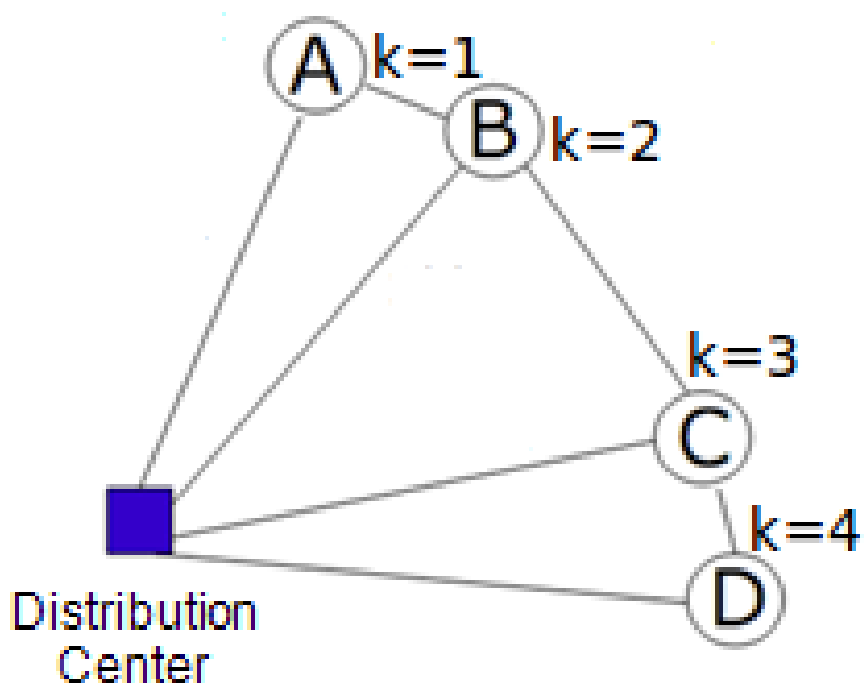
# Problem Statement

- The company owns a distribution center which is resupplied every three days to meet the delivery demands over that period.
- Holding product in the distribution center has a daily per unit cost.
- The company delivers the product to its customers through a truck fleet. The trucks are all the same and have a maximum capacity.
- Each truck can only go out one delivery route each day. However multiple trucks can go on the same route.

# Problem Statement

- Various tours through the network of possible paths to customers are available to deliver the product to the customers.
- The company wants to establish a three day schedule that would meet the daily demand and respect the inventory limits of each customer.
- The company seeks to minimize the inventory and transportation costs.

# 2. Variables and notations

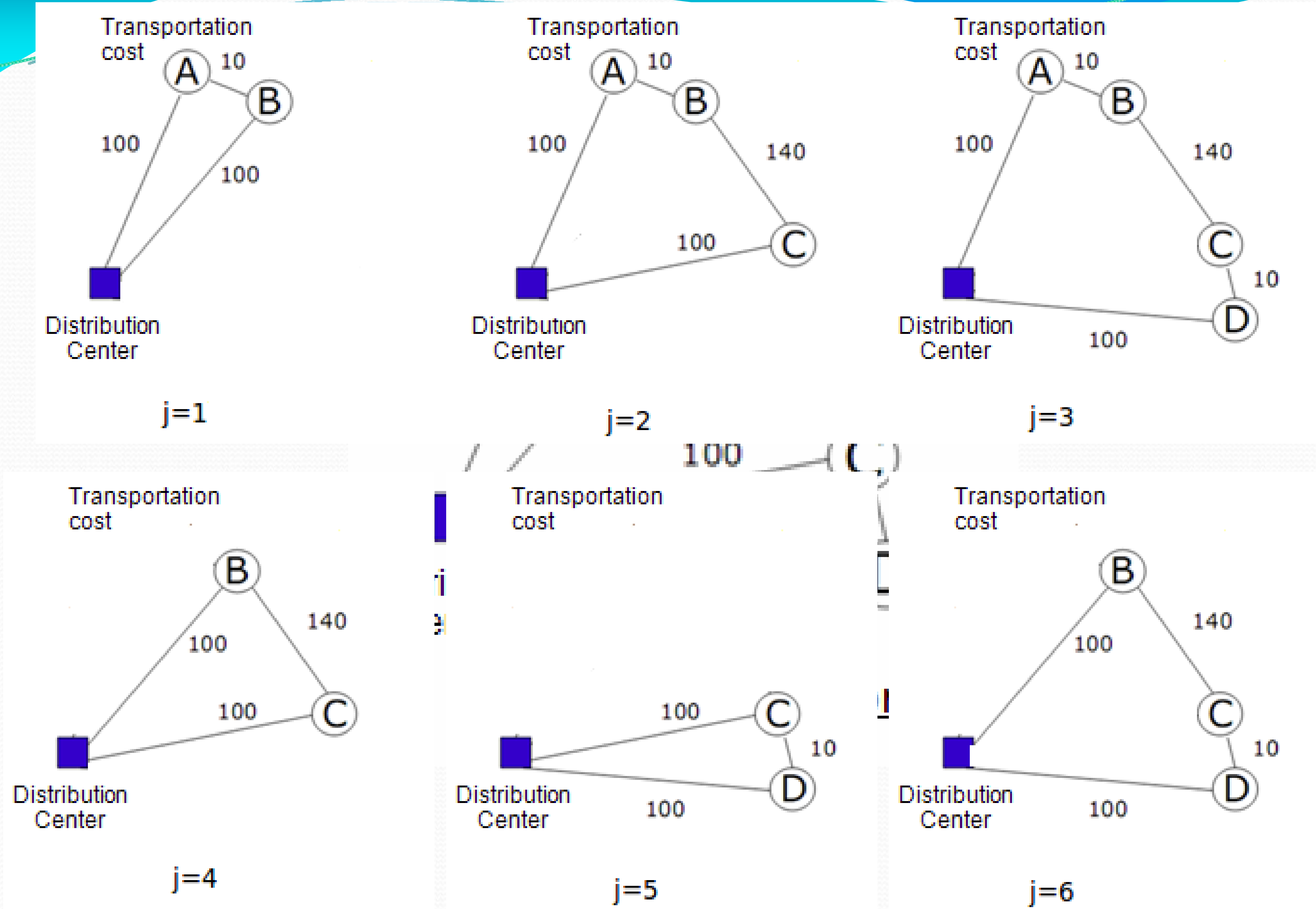



**Transportation Network**

- **2.1 Subscripts & superscripts**
- *i*: the index of days;

*The i-th day of the period;*

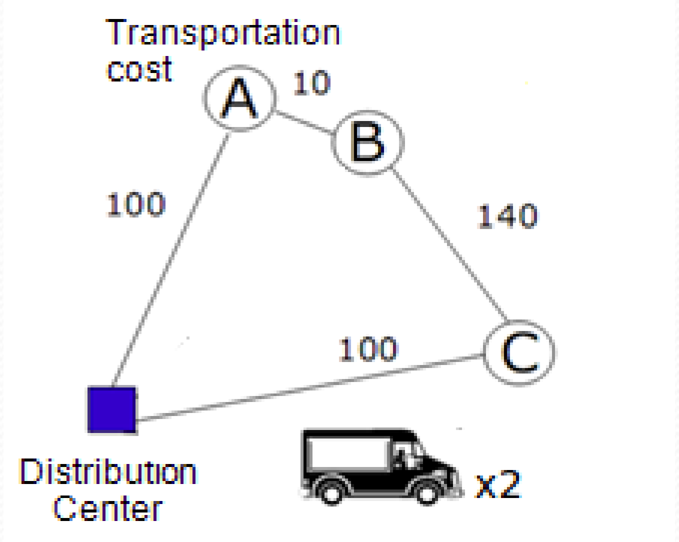
- *k*: the index of customers;
- *j*: the index of routes;



Transportation cost
A
B
10
100
100
Distribution Center
j=1
Transportation cost
A
B
C
10
100
140
100
Distribution Center
j=2
Transportation cost
A
B
C
D
10
100
140
10
100
Distribution Center
j=3
Transportation cost
B
C
140
100
100
Distribution Center
j=4
Transportation cost
C
D
100
10
100
Distribution Center
j=5
Transportation cost
B
C
D
140
100
10
100
Distribution Center
j=6

# 2.2 Variables

- $x_{ij}$: The number of trucks sent out on the *i*-th day to deliver products by the *j*-th route.
- $c_j$: the transpotation cost of the *j*-th route.
- $a^k_{ij}$: The units of products delivered to the *k*-th coustomer on the *i*-th day by the *j*-th route.



# Objective Function

- Transportation Cost
- $C_T = \sum_{i}^{3} \sum_{j}^{N} x_{ij} c_j$ ; $N$ indicates the total number of route
- Inventory Cost

$$C_I = c_{inv}(A - \sum_{k}^{M} \sum_{j}^{N} a_{1j}^{k})$$

$$+ c_{inv}(A - \sum_{k}^{M} \sum_{j}^{N} a_{1j}^{k} - \sum_{k}^{M} \sum_{j}^{N} a_{2j}^{k})$$

$$+ c_{inv}(A - \sum_{k}^{M} \sum_{j}^{N} a_{1j}^{k} - \sum_{k}^{M} \sum_{j}^{N} a_{2j}^{k} - \sum_{k}^{M} \sum_{j}^{N} a_{3j}^{k})$$

- M indicates the total number of customers; N indicates the total number of routes; A indicates the total amount of products in distribution center and $c_{inv}$ indicates the inventory cost every day

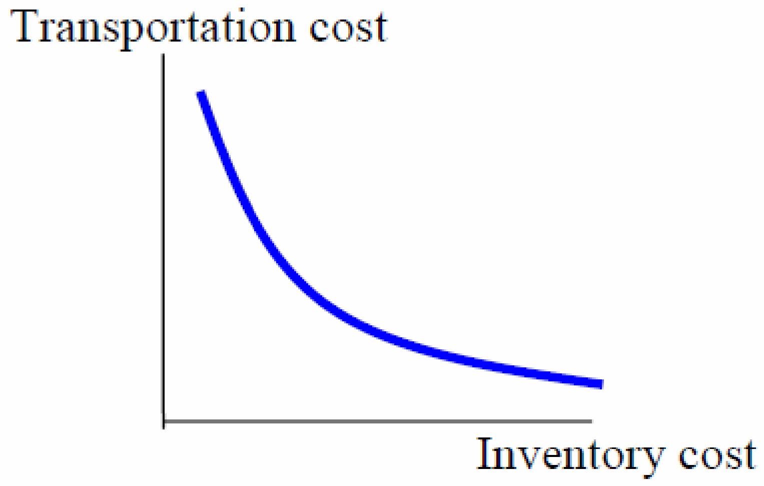
# Objective Function

- Total Cost

$$C = C_I + C_T$$

- Objective Function

$$\min C = C_I + C_T$$




Relationship between transportation and inventory cost

# Review

- The system consists of a distribution center and several customers. The distribution center delivers multiple items to customers on a daily basis. The products are supplied with order up to level inventory policy which vehicles deliver the amount for the difference between the on-hand inventory and the maximum inventory level. The shipment planning period is 3 days.

# Constraints

- The number of vehicles is limited.
- Vehicles on route j can't send products to customers which are not on route j.
- The amount of products sent on route j can't exceed the vehicles' ability.
- The products amounts delivered can not exceed the maximum inventory level. And the products sent should satisfy the daily consumption of customers.

# Constraints-1

- The company operates by one fleet type with a limited number of vehicles.
- So we add the constrain in order to make sure that the number of vehicles sent out each day should be smaller than the total number of vehicles.
- N -> the available amounts of vehicles

$$\sum_{j} x_{ij} \leq N$$

- i -> day ; j -> route;
- $X_{ij}$ ->the number of vehicles sent out on route j day i.

# Constraints-2

- For each route j, it passes certain customers. The vehicles which go through route j can only send products to the customers which are on route j.  This means that if customer $k$ are not on the route j, the number of products sent to customer k should be zero.
- If customer $k$ is not on route j, then

$$\forall i,j,k \quad a_{ij}^{(k)}=0$$

# Constraints-3

- The shipment quantity cannot exceed vehicle’s capacity. So the products sent at certain route should be smaller than the vehicles’ capacity.
- P –> the capacity of vehicles

$$\sum_k a_{ij}^{(k)} \le x_{ij} * P$$

- $\sum_k a_{ij}^{(k)}$ ->all the products sent on route j day i
- $x_{ij} * P$ -> the total capacity on route j day i

- A deterministic demand is considered in this study. Each customer has different consumption rate per day. And the customers' capacity of storage is given. The products sent to customers can't not exceed the storage.
- So, each time when delivering the products, make sure that the amount of each delivery does not cause the capacity of the customer to be exceeded. And the customers should have at least the daily required products on any day.

- For example, if customer k 's consumption is 100 per day, and capacity of storage is 500.
- By day 1, the amount of products sent should be larger than 100, and smaller than 500;
- By day 2, the amount of products sent should be larger than 200, and smaller than 600;
- The min products sent to customer k by day i:

$$\min_{ik} = consu_k * i$$

- The max products sent to customer k by day i:

$$\max_{ik} = capac_k + consu_k * (i - 1)$$

# Constraints-4

- Considering the customers' capacity. We need to constraint how many products can be sent to costumer per day. So we have,

$$\forall k,i \qquad \min_{ik} \le \sum_{r=1}^{i} \sum_{j} a_{rj}^{(k)} \le \max_{ik}$$

- k -> the index of costumers ;
- $\sum_{r=1}^{i} \sum_{j} a_{rj}^{(k)}$ -> the products sent to customer k by day i;
- $\min_{ik}$ ->the min products sent to customer k by day i ;
- $\max_{ik}$ ->the max products sent to customer k by day i.

# Data Required

- Transportation Network. Including topology structure and costs of each branch.
- List of customers.
- Capacity and demand of the customers.
- Number of trucks and their capacity.
- Inventory cost.
- Initial amount of product in distribution center inventory.

# *Any Questions?*