Graph Theory
Assignment 5
Due 10:00 AM, Friday February 10th.

1) Prove that if some equivalence class in the transitive closure of the *in a* P_3 relation does not span V then G has a homogeneous set.

(2) Use (1) to prove that if G has no homogeneous set and has more than two transitive (possibly cyclic) orientations then it contains a triangle two of whose edges are in one equivalence class and the third edge of which is in a second.

(3) Prove that if S is a maximal subset of $N(x)$ such that some equivalence class spans S (i.e. there is a spanning tree for S in this equivalence class) and none of the edges from S to x are in this equivalence class then provided S has at least two vertices, S is a homogeneous set of G

(4) Prove that if G has no homogeneous set then its edges are all in the same equivalence class and hence if it has a transitive orientation it has exactly two of them, both of which are acyclic.

(5) Prove that if G is connected and connected in the complement then its maximal homogeneous sets are disjoint and the subgraph obtained by taking one vertex from each such set contains no homogeneous set.

(6) Prove that if $\overline{G}$ is disconnected with components $U_1, ..., U_k$ then it has exactly

$$k! \prod_{i=1}^{k} n_i$$

acyclic tranistive orientations, where n_i is the number of acyclic transitive orientations of $G[U_i]$.

(7) Give a polynomial time algorithm to count the number of acyclic transitive orientations of a graph.