Graph Theory
Assignment 2
Due 10:00 AM, Friday February 3rd.

1) (a) Prove that if we have a non-negative real weight vector on the edges of a graph G satisfying:

$$\forall v \in V, \quad \sum_{v \in e} x_e = 1$$

and

$$\forall S \subseteq V, |S| \ odd : \qquad \sum_{e \in E[S, V-S]} x_e \geq 1$$

then G has a perfect matching.

(b) Prove that every 2-edge connected 3-factor has a perfect matching.

2) (a) Show that the maximum weight of a fractional matching:

$$max \quad \sum_{e \in E} x_e$$

$$s.t. \quad \forall v \in V, \quad \sum_{v \in e} x_e \leq 1$$

$$x \geq 0$$

is at most $\frac{3}{2}$ times the size of a maximum (integer) matching.

3) Use Tutte's f-factor theorem to prove that every 2d-factor can be partitioned into d 2-factors.

4) Use Tutte's f-factor theorem to prove that every 3 edge connected 2d-1 factor can be partitioned into a d-factor and a $(d-1)$-factor.

5) (a) Prove that a graph in which the vertices of maximum degree form a stable set has a Δ edge colouring.
(b) Extend this to graphs where the vertices of maximum degree form a tree.