Assignment 1

Due Wednesday January 11, 2005, 10:O0 AM.

1) Find for each k, a graph G_k which has $4k+2$ vertices, $4k+2$ edges, one perfect matching and more than 2^k near-perfect matchings.

2) Prove that a graph has an orientation in which every vertex has indegree at most k if and only if it does not contain a subgraph H satisfying $|E(H)| > k|V(H)|$.

3) Give a polynomial time algorithm to determine if G contains an orientation as in 2.

4) Use Posa's flipping technique to prove that for any graph G on $n > 3$ vertices, if every vertex of G has at least $\frac{n}{3} + 4$ neighbours, and together every two vertices see at least $\frac{2n}{3}$ other vertices then G has a Hamilton cycle.