Heap building bounds

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We have considered the uppper and lower bounds for building heaps in the worst and average cases. Heaps are binary tree for which all its levels are full except the last one. The nodes on the last level are all as much to the left as possible. A max-heap has the property that every node has value greater or equal to that of its children. We have studied the contruction of heaps in the comparison model. In this model, at each step, an algorithm chooses two elements and compares to determine which is bigger.

We have shown that two of the previous results on the subject to be incorrect. We have proven that the analysis of the average case algorithm given in [2] is incorrect and that this algorithm is actually worse than that shown in [4]. We have shown that the adversary described in [1] does not yield its claimed bound and that its lower bound is worse than that of an information theory argument given in [3]. We have constructed an adversary which yields a better bound than that in [3]. The analysis of our adversary is formulated as a linear program. This linear program, generated by a computer program, consists of 4990 variables, 209 constraints and totals 70 pages.

References

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[4] C. J. McDiarmid and B. A. Reed. Building heaps fast. *J. Algorithms*, 10(3):352–365, 1989.