Stochastic Processes

We consider random variables which are the cumulative effect of a sequence of random choices. Thus, we have a sequence $T_1, ..., T_n$ of random choices and an associated sequence $S_0, S_1, ..., S_n$ of random variables where S_n is a function of $T_1, ..., T_n$.

For example, we may consider a sequence of one dollar wagers on the toss of a fair coin, where S_n is the net winnings after n wagers. In this example, S_n is the sum of n i.i.d random variables each of which is 1 with probability 1/2 and -1 with probability 1/2. More generally, a *Random Walk* in $\mathcal{Z}^d$ consists of a sequence $X_1, .., X_n$ of i.i.d vectors in $\mathcal{Z}^d$ where our current position S_n is the sum of X_1 to X_n.

We will also consider random walks where the step X_i we take depends on our current position S_{i-1}, so called *Markov Chains*. This allows to, e.g., account for currents when considering the motion of a particle in a fluid. It also allows us to model the flow through many complex random systems where we move between various states (which we can think of as integers) , and greatly extends the reach of this theory.

We can also consider sums of real-valued random variables. For example, if the X_i are non-negative i.i.d random variables each of which represents the lifetime of a lightbulb, and we assume instantaneous replacement then S_n is the length of time the first n lightbulbs lasted. *Renewal Theory* deals with sums of i.i.d non-negative real variables.

We can also introduce a continuous model of time into our Markov Chain theory. For example, we may generate a random time T_i that it takes to take the step from state S_{i-1} to state S_i. *Birth and Death Processes* are examples of such *Continuous-Time Discrete State Space Markov Chains*.

Additional topics which we will cover if time permits: Brownian Motion, Percolation, Markov Decision Models and Hidden Markov Models.

In studying these processes, we will need to learn about *generating functions*, *moment generating functions*, *Concentration Inequalities*, and *coupling techniques*.