Assignment 2

Due 11:00 March 18th

1) Problem 29 on page 86 of the text.

2) Show that for any branching process, the probability that Z_n lies between 2 and n goes to zero as n goes to infinity.

3)
Consider a Markov Chain with state space

$$\Omega = \mathcal{Z} \cup \{*\}$$

where

$$p_{i,j} = 0 \ if \ |i-j| \neq 1$$

$$\forall i, \ p_{i,i+1} = p_{i,i-1} = \frac{1}{4}$$

$$\forall x, \ p_{x,*} = \frac{1}{2}$$

$$p_{*,0} = 0, \forall i \neq 0 \ p_{*,i} = \frac{1}{4A|i^3|}$$

for $A = \sum_{i=1}^{\infty} \frac{1}{|i^3|}$.

Determine if each state in this chain is transient, null recurrent or positive recurrent. Does the chain have a limit distribution π ? If so give as good upper and lower bounds on $\pi(x)$ for each $x \in \Omega$ as you can.

4)
Consider a Markov Chain with state space

$$\Omega = \mathcal{Z} \cup \{*\}$$

where

$$p_{i,j} = 0 \ if \ |i-j| \neq 1$$

$$\forall i \neq 0, \ p_{i,i+1} = p_{i,i-1} = \frac{1}{2} - \frac{1}{2|i^3|}$$

$$\forall i \neq 0, \ p_{i,*} = \frac{1}{|i^3|}$$

$$p_{0,1} = p_{0,-1} = \frac{1}{2}$$

$$p_{*,*} = p_{*,0} = 0, \forall i \neq 0 \ p_{*,i} = \frac{1}{2A|i^3|}$$

for $A = \sum_{i=1}^{\infty} \frac{1}{|i^3|}$.
Determine if each state in this chain is transient, null recurrent or positive recurrent. Does the chain have a limit distribution π ? If so give as good upper and lower bounds on $\pi(x)$ for each $x \in \Omega$ as you can.

5)
Consider a Markov Chain with state space

$$\Omega = \mathcal{Z} \cup \{*\}$$

where

$$p_{i,j} = 0 \ if \ |i-j| \neq 1$$

$$\forall i > 0, \ p_{i,i+1} = \frac{2}{3} - \frac{1}{4|i^3|}$$

$$\forall i > 0, \ p_{i,i-1} = \frac{1}{3} - \frac{1}{4|i^3|}$$

$$\forall i < 0, \ p_{i,i+1} = \frac{1}{3} - \frac{1}{4|i^3|}$$

$$\forall i < 0, \ p_{i,i-1} = \frac{2}{3} - \frac{1}{4|i^3|}$$

$$\forall i \neq 0, \ p_{i,*} = \frac{1}{2|i^3|}$$

$$p_{0,1} = p_{0,-1} = \frac{1}{2}$$

$$p_{*,*} = p_{*,0} = 0, \forall i \neq 0 \ p_{*,i} = \frac{1}{2A|i^3|}$$

for $A = \sum_{i=1}^{\infty} \frac{1}{|i^3|}$.
Determine if each state in this chain is transient, null recurrent or positive recurrent. Does the chain have a limit distribution π? If so give as good upper and lower bounds on $\pi(x)$ for each $x \in \Omega$ as you can.

6)
Consider the following two Markov Chains both with state space:

$$\Omega = \{1, 2, .., 2n\}$$

and $p_{i,j} = 0$ if $i + j$ is even and $i \neq j$. The first chain has $p_{i,i} = \frac{1}{2}$ for all i and $p_{i,j} = \frac{1}{2n}$ for all i and j whose sum is odd. The second chain satisfies (A) $p_{i,j} = \frac{1}{n}$ for all i and j whose sum is odd unless i is $2n$ in which case $p_{i,j} = \frac{1}{2n}$, and (B) $p_{i,i} = 0$ unless $i = 2n$ in which case $p_{i,i} = \frac{1}{2}$. Give as good a bound as you can on the mixing times of these two chains.