1 Some Sample Problems

We discussed the following problems:

TSP
Input: An integer n, distances $d_{i,j}$ for $1 \leq i \neq j \leq n$ with $d_{i,j} = d_{j,i}$.
Search Goal: The permutation π which minimizes

$$\sum_{i=1}^{n-1} d_{\pi^{-1}(i)\pi^{-1}(i+1)} + d_{\pi^{-1}(n)\pi^{-1}(1)}.$$

HAMILTON CYCLE
Input: A Graph G
Search Goal: A Cycle of G passing through all of its vertices.

LONGEST PATH
Input: A Graph G, two vertices x and y of G.
Search Goal: A path of G between x and y which maximizes the number of edges.

SHORTEST PATH
Input: A Graph G, two vertices x and y of G.
Search Goal: A path of G between x and y which minimizes the number of edges.

Definition: A matching is a set of disjoint edges.

PERFECT MATCHING
Input: A Graph G with an even number of vertices.
Search Goal: A matching of size $\frac{|V(G)|}{2}$ in G.

We noted that Hamilton cycle could be solved by solving a TSP where every $d_{i,j}$ is either 0 or 1. Thus, TSP is at least as hard as HAMILTON CYCLE.

We noted that HAMILTON CYCLE could be solved by solving LONGEST PATH for every edge xy of G. Thus, HAMILTON CYCLE cannot be much more difficult than LONGEST PATH.

We remarked that SHORTEST PATH could be solved quickly even though it looks very similar to LONGEST PATH for which we have no efficient algorithm. In the same vein, although PERFECT MATCHING is similar to HAMILTON CYCLE, we have a fast algorithm for the first and no fast algorithm for the second.

As an aside we mentioned the first graph theory problem, studied by Euler in 1736:

Eulerian Tour
Input: A Graph G
Search Goal: A closed walk of G passing through each edge exactly once.

Interested students could try and develop an algorithm for solving this problem.