Some Solutions

1) For each vertex v and integer i between 1 and k, we have a (Boolean) variable $x_{v,i}$ which is an indicator variable for the event that v is coloured i. We need to following clauses:

   For every vertex v we have $x_{v,1}$ OR $x_{v,2}$ OR …OR $x_{v,k}$

   For every vertex v and $1 \le i < j \le k$ we have: (NOT $x_{v,i}$) OR (NOT $x_{v,j}$)

   For every edge uv and colour i we have: (NOT $x_{u,i}$) OR (NOT $x_{v,i}$)

   Given a k-colouring of G, we obtain a satisfying truth assignment by setting $x_{v,i}$ to be true if v is coloured i and to be false otherwise. Given a satisfying truth assignment, the first set of clauses above ensure that for each v there is at least one i such that $x_{v,i}$ is true. The second set of clauses ensure that there is exactly one such i for each v. Finally the third set of clauses show that if we give each vertex v the colour i for which $x_{v,I}$ is true then we obtain a k-colouring of G.

2) Given an instance of Satisfiability consisting of a family F of clauses and a set X of variables, we construct an instance of 3-SAT whose set of variables is the union of X and for every clause C in F containing $j=j_C > 3$ literals $z_c^1,\ldots,z_c^j$. for each i with $2 \le i \le l-2$ a literal (NONE OF THE FIRST i LITERALS OF C ARE TRUE). The family of clauses of the 3-SAT instances is the union of a set of clauses for each clause C in F. If $j_c = 3$, this set of clauses is just C itself. If $j_c < 3$ then it is the clause obtained from C by adding $3-j_C$ copies of the last literal. Otherwise the 3-sat instance has the following clauses corresponding to C:

   z_c^1 OR z_c^2 OR (NONE OF THE FIRST 2 LITERALS OF C ARE TRUE)
   z_c^l OR z_c^{l-1} OR (NOT (NONE OF THE FIRST l-2 LITERALS OF C ARE TRUE))
   For i between 3 and l-2:
   (NOT (NONE OF THE FIRST i-1 LITERALS OF C ARE TRUE)) OR
   z_c^i OR (NONE OF THE FIRST i LITERALS OF C ARE TRUE)

   Given a satisfying truth assignment for the Satisfiability instance, we obtain a satisfying assignment for the 3-SAT instance as follows. We use the same assignment on X, and for each other variable we assign it the value that this assignment gives its label.

   Given a satisfying truth assignment for the 3-SAT instance, its restriction to X yields a satisfying truth assignment for the Satisfiability instance.

3) Given a graph G we construct a graph G' whose vertex set is the union of V and V'={v'|v is in V}, and whose edge set is the union of {v'u, u'v | uv in E}. We showed that the objective function value of an optimal solution to the fractional matching problem in G' is precisely twice that for the optimal solution to the fractional matching problem in G as follows . Given a fractional matching in G', we obtain a fractional matching in G by setting

$x_{uv}=(x_{u'v}+x_{v'u})/2$. Given a fractional matching in G, we obtain a fractional matching in G' by setting $x_{u'v}=x_{uv'}=x_{uv}$.

4) We had the following variables:
Input variables: $x_{i,j}$ for I between 1 and n and j in {0,1,2}.
Finding the 2s variables:
For every i and j with $1 \le i < j \le n$, $The2sare_{i,j}$
Carrybitvariables c_1 to c_n

We have the following clauses:

Input assignment clauses:
For $1 \le i \le n$ and j in {0,1,2}: $x_{i,j}$ if $j=a_i$ and NOT $x_{i,j}$ otherwise.

Choosing the 2s clauses:
A clause which is the or of $The2sare_{i,j}$ over all (i,j) with $1 \le i < j \le n$.
For every $1 \le i < j \le n$ and $1 \le k < l \le n$ with (i,j) not equal to (k,l):
(NOT $The2sare_{i,j}$) OR (NOT $The2sare_{k,l}$)
For every $1 \le i < j \le n$ and $1 \le k \le n$ with k not in {i,j}
(NOT $The2sare_{i,j}$) OR NOT ($x_{k,2}$)
(NOT $The2sare_{i,j}$) OR $x_{i,2}$
(NOT $The2sare_{i,j}$) OR $x_{j,2}$

Verifying the sum clauses. We need some clauses for every choice (i,j) of the position of the 2s. To b finalized next class.