Lecture 5: We continued our study of the Simplex Method for solving LPs.
As set out in the last lecture, the main steps of this method are:

1) Find a feasible dictionary.
2) Determine if there is a non-basic variable with positive cost coefficient in the expression for z. If not, terminate the current solution is optimal
3) Choose a non basic variable x_e with positive cost coefficient in this expression (x_e is the entering variable).
4) If there is no variable which decreases if we increase x_e while leaving the other non-basic variables at zero then, the problem is unbounded; terminate and return this fact. Otherwise, find a non-basic variable x_l whose nonnegativity gives the most stringent upper bound on the possible values of x_e if we increase it while leaving all other nonbasic variables at 0. x_l is the leaving variable.
5) Rewrite the equation for x_l in the dictionary as an equation for $x_{e.}$
6) Replace x_e by the RHS of this equation in all the other equations of the dictionary.
7) Go back to Step 2.

We proved (see page 33 of the text) that each choice of a basis yields a unique dictionary.It follows that there are at most $\binom{n+m}{m}$ different dictionaries. Thus, for any LP in standard form, provided we can find a feasible dictionary in Step 1, and that we never visit a dictionary twice, then the algorithm will either terminate with an optimal solution in Step 2 or because the problem is unbounded in Step 4.
We next noted that the objective function can never decrease in an iteration and that it remains the same precisely if the leaving variable had value zero in the dictionary at the beginning of the iteration. We called such iterations *degenerate* and dictionaries with a zero-valued basic variable *degenerate*. We observed that the only way that we could visit a dictionary twice was via a cycle consisting of a sequence of degenerate iterations. This involves staying at the same solution which has more than n zero valued variables.
We discussed the example on pages 31-32 of Chavatal where such a cycle is traversed.
We noted that the intuitition behind the lexicographic method for avoiding degenerate iterations and hence cycle was that it perturbs the dictionary by modifying the RHS of each of the inequalities very slightly so that no basic solution with more than n zero valued variables exists. I noted that one of my motivations for mentioning the perturbation method is that I wanted to stress that one reason a problem may be difficult to solve in the worst case is because of some denegenerate situation and that if we can somehow avoid this degeneracy, this may allow us to solve the problem more efficiently. We did not discuss the details of the method nor are you responsible for them.
Instead, we followed the proof in the text that if from amongst all choices for the entering variable we always chose that with the lowest subscript and from amongst all choices for the leaving variable we always choose the one with the lowest subscript then we would never repeat a dictionary and hence would eventually terminate provided we could find a feasible dictionary in Step 1.
I announced that I had put up some exercises on the website for the next class.