Lecture 2: We finished off formulating the 5 examples from the Exercise Set as Linear Programs. We noted that the size of the LP for MWST could be very large in terms of the input graph. We gave a second smaller formulation exploiting the fact that a graph has no cycles precisely if we can relabel its vertices as $v_1,\ldots,v_n$ so that v_i sees v_j for at most one $j<i$.

We stated the Max-Flow Min Cut theorem and noted that there was an algorithmic proof which yielded an efficient algorithm to solve the Integer variant of this LP provided all capacities are integer. We discussed reductions to Maximum-Flow Minimum Cut and gave as an example the reduction of the open pit mining problem, which is found on page 373 of Chvatal. Note that Chapter 22 of Chvatal and Chapter 7 of Kleinberg-Tardos discuss a variety of diverse problems which can be reduced to/formulated as Maximum Flow problems. You should be able to follow pages 369-371 of Chvatal which define a max flow problem and state the theorem, except for the proof of the theorem which you need not worry about.

We defined the Fractional Relaxation of an IP (the LP obtained by dropping the constraint that the variables must be integers). We discussed Fractional Matching and noted a triangle had a fractional matching of size 3/2 but its largest matching was of size 1. We showed that fractional matching has the same solution as matching for bipartite graphs using the max matching/min cover theorem which we deduced from the max flow min cut theorem. . This is a nice property of reducing to Maximum Flow problems. For such problems the IP and its fractional relaxations always have the same solutions which make the IPs easy.