Lecture 13: Cutting Stock I

We considered the Cutting Stock Problem discussed in Chapter 13 of Chvatal. Thus we have raw rolls of width r, and we want to cut them up to obtain b_i rolls of width w_i for i=1 to m. For some n, there are n possible cutting pattern. The j^{th} pattern produces $a_{j,i}$ finals of width w_i for some choice of $a_{j,i}$ such that:

$$\sum_{i=1}^{m} a_{j,i}\, w_i \leq r$$

Letting x_j be the number of raw rolls we cut using the j^{th} pattern. We are led to the Integer Program,

min $\sum_{j=1}^{n} x_j$ subject to:
for all i between 1 and m: $\sum_{j=1}^{n} a_{j,i}\, x_j \geq b_i$
where the x_j must be nonnegative integrals.

Having obtained an optimal solution to the LP we can obtain a feasible solution to the IP within m of optimal by rounding up the x_j or rounding them down and satisfying residual demand.

Solving the LP may be difficult as the number of cutting patterns can be exponentially large in terms of the input. For this reason, we work with at most m (basic) cutting patterns at any given time. Having found the optimal solution (and optimal dual solution) for some set of cutting patterns we attempt to generate a non-basic cutting pattern such that the corresponding primal variable has a positive cost coefficient in the LP in standard form obtained by multiplying the objective function value and all the constraints by -1.

If we add cutting pattern j, then we obtain a dictionary for the new larger standard form LP with slack variable x_{n+i}' corresponding to the i^{th} equation from one for the original standard form LP by substituting x_{n+i}' $-a_{j,i}x_j$ for x_{n+i} and subtracting x_j from the objective function. Considering an optimal dual solution for the small LP (possibly read of the last row of an optial dictionary) and letting y_i be the value of the dual variable corresponding to the i^{th} equation, since the coefficient for x_{n+I} in the objective function is $-y_i$, we obtain that the coefficient of x_j in the expression for the new objective function is $-1+\sum_{i=1}^{m} y_i\, a_{j,i}$. So a cutting pattern has a positive cost coefficient in the objective function precisely if $\sum_{i=1}^{m} y_i\, a_{j,i}>1$. Generating such a cutting pattern boils down to finding a_i such that $\sum_{i=1}^{m} y_i\, a_i>1$ and $\sum_{i=1}^{m} w_i\, a_i \leq r$.

This is a multi-knapsack problem with W=r, V=1 and $v_i = y_i$. We will discuss how to solve it next class. We went through the material on pages 195-200 of Chvatal. Note that he is now using a more advanced way of solving LPs without putting them in standard form. So we had to take the duals and verify that the y he claimed was an optimal dual solution actually was ourselves.