Lecture 6: Duality

We defined the dual of a linear program. We saw that the objective function value of any feasible solution to the dual yields an upper bound on the objective function value of any feasible solution to the primal. We proved the duality theorem of linear programming, that if the primal has an optimal solution then so does the dual and their solution values are equal. We did so by considering the feasible dual solution which can be read off of the cost coefficients of an optimal dictionary for the primal problem. We covered the material in Chapter 5 of Chvatal from the beginning of the Chapter to the middle of page 62.

We took the dual of the fractional matching polytope and saw that it was

Min $\sum_{v \ in\ V} y_v$ subject to for all edges e=uv, $y_u+y_v \geq 1$and for all v in V, $y_v \geq 0$.

We note that if we added integrality constraints (i.e. insisted that the y_v were integer) then we obtained an ILP for the minimum cover problem.

I explained that one approach to developing algorithms to solve problems is to formulate them as ILPs and then take the dual of the fractional relaxation so as to develop an understanding of what is the (dual) structure that prevents us from getting a good solution.