COMP362 Honours Algorithm Design Assignment 4

Due at 14;30 on Thursday April 14th 2016.

1. Given a set S of n points in the plane, define the depth of a point $p \in S$ to be the number of convex hulls that need to be peeled (i.e. removed) for p to be a vertex of the convex hull. Design a $O(n^2)$ time algorithm that finds the depth of all points of S.

2. The *distinctness* problem asks: Given a list of elements $x_1, x_2, \ldots, x_n$, are there two elements x_i and x_j that are the same ($x_i = x_j$)?

   (a) How would you use sorting to solve the distinctness problem? What is the running time your this algorithm?

   (b) Suppose you have a $\Omega(n \log n)$ lower bound on sorting. Would your reduction in the previous section give you a lower bound for the distinctness problem? Explain.

3. You are given a set P of n points in the plane incrementally, one at a time. You would like to compute the convex hull of the points given so far, after receiving each point. Clearly, running Graham's scan every time you receive a point will work, but your running time in this case will be $O(n^2 \log n)$. Design an algorithm that finds the convex hull of the given points incrementally in a total of $O(n^2)$ time.

4. Recall that Chan's algorithm for computing the convex hull uses doubly exponential search to "guess" the value h of the number of points on the convex hull. So at the i-th iteration of the algorithm starts by splitting the pointset into groups of $m = 2^{2^i}$. The iterations continue until the gift wrapping step finds the hull with $m = h$ vertices. In each of the following cases for m, indicate what the running time of Chan's algorithm would be and explain your answer.

   (a) $m = i^2$

(b) $m = 2^i$

(c) $m = 2^{2^i}$