MORE REDUCTIONS

1) The Subset sum problem is defined in section 6.4 of Kleinberg and Tardos. In the decision version of the problem we are given integers k, W, and $a_1,\ldots,a_n$ and asked if we can find a subset of the integers $a_1,\ldots,a_n$ whose sum is at least k and at most W. A special case is the **Splitting The Sum In Half Problem.** An instance consists of integers $a_1,\ldots,a_n$. We are asked if we can partition {1,…,n} into two sets B and C so that:

$$\sum_{i \in B} a_i = \sum_{i \in C} a_i = \frac{1}{2}\sum_{i=1}^{n} a_i$$

Show that the Subset Sum decision problem can be polynomially reduced to the Splitting The Sum In Half Problem, and vice-versa.

2) An instance of the **Splitting Many Sums In Half Problem** problem consists of an integer k, and, for some integer n, a set of n ordered k tuples $(a_1^1,a_1^2,\ldots,a_1^k)$, $(a_2^1,a_2^2,\ldots,a_2^k)$, $(a_n^1,a_n^2,\ldots,a_n^k)$, We are asked if we can partition {1,…,n} into two sets B and C so that for each j between 1 and k we have:

$$\sum_{i \in B} a_i^j = \sum_{i \in C} a_i^j = \frac{1}{2}\sum_{i=1}^{n} a_i^j$$

Show that the Splitting Many Sums in Half Problem can be polynomially reduced to the Splitting The Sum in Half Problem and vice versa.

3) Show that 3-SAT can be polynomially reduced to the Splitting Many Sums In Half Problem. (Hint: the k in the Splitting Many Sums In Half Instance corresponding to an instance of 3-SAT is the number of variables plus the number of clauses in that 3-SAT instance.)