Lecture 6: Duality

We defined the dual of a linear program, by considering the example presented at the start of Chapter 5 of Chvatal. We proved the duality theorem of linear programming by considering the feasible dual solution which can be read off of the cost coefficients of an optimal dictionary for the primal problem, as it is done in Chapter 5 of Chvatal.
We noted that this dual solution is a certificate of length O(n) which can be used to verify the optimality of the solution in O(nm) time.
We will discuss duality further next week. I asked you to take the dual of the fractional matching LP in preparation.

We discussed 2-trees. As an example we considered triangulatons of convex polygons.

A graph G with vertex set V and edge set E is a 2-tree if we can find a rooted tree (T,r) such that:

(i) For every node s of T, there is a set of three vertices of a_s,b_s,c_s which induces a triangle in T (i.e. a_sb_s,b_sc_s, and a_sc_s are all edges of G),
(ii) For distinct s and t, $a_s \neq a_t$, furthermore a_s is neither b_r nor c_r.
(iii) For every node s of T other than the root r, letting p be the parent of s, we have: $\{b_s,c_s\}$ is a subset of $\{a_p,b_p,c_p\}$, and
(iv) Every edge of G is in one of the triangles corresponding to the nodes of T.

I asked you to try and find a sketch of a proof that every 2-tree is planar and to provide a polynomial (or even better linear) time algorithm to determine if an input graph is a 2-tree. We will discuss this on Monday.