Discrete Optimization II COMP 567

Homework 3 Due: Friday April 18, 2008, email pdf file to: avis@cs.mcgill.ca

1. Consider the variables $x_{ij}, 1 \le i < j \le n$ defined on the edges of a complete graph on n vertices. (Assume x_{ij} is identical to x_{ji} for convenience.) We have two ways of writing subtour elimination constraints for the minimum spanning tree problem:

$$\sum_{i,j \in S} x_{ij} \le |S| - 1 \quad \text{for } all\ S \subset \{1, 2, \ldots, n\}. \tag{1}$$

$$\sum_{i \in S, j \in \bar{S}} x_{ij} \ge 1 \quad \text{for } all\ S \subset \{1, 2, \ldots, n\}, \tag{2}$$

where $\bar{S}$ is the complement of S. These constraints are combined with

$$\sum_{1 \le i < j \le n} x_{ij} = n - 1$$

to get a formulation for the MST.
(a) Show that (1) and (2) are equivalent for 0/1 solutions (ie the solutions correspond to trees in the complete graph).
(b) Show that (1) and (2) are **not** equivalent for fractional solutions. Ie. show one condition implies the other, but not vice versa.

2. This exercise is related to facets of the Knapsack polytope. Read Wolsey, Section 9.3. Now do exercise 4, P. 162. (Hint for (ii) and (iii): use lrs to generate vertices of the respective regions, and select a set of affinely independent 0/1 vertices that lie on the given facets. Ie. Approach 1 on P. 144.)

3. Consider a farmer who has 500 acres and has to decide how much land to devote to each of the three crops: wheat, corn and sugar beets. He knows that he needs 180T of wheat and 260T of corn to feed his cattle. These amounts can be raised or purchased. Any production in excess can be sold at a price 160$ /T for wheat and 140$ /T for corn. The purchase price is 40% higher than the selling price. The farmer can sell sugar beet at a price 36$ /T. However, for sugar beet that is in excess of the quota he can only sell for 10$ /T. His quota is 6000. The planting cost is 150$, 230$ and 260$ per acre for wheat, corn and sugar beets respectively. On average, the per acre yields are 2.5T, 3T and 20T for wheat, corn and sugar beets.

With probability 35%, it will be a good weather next year, which means all yields are 125% of the normal ones and the selling/purchasing price of wheat/corn is 90% of the normal ones. With probability 25%, it will be bad weather, which means a 75% yield for all crops, and 110% of the selling/purchasing price of wheat/corn. With 40% of probability, the weather is normal and all coefficients are equal to those given above. (Note that the price for sugar beet remains the same for all three scenarios)

Formulate this problem into a stochastic linear problem, in both extensive form and concise form.Solve the LP using CPLEX.