# Integer Programming 308-567B
# Class Test March 7, 2002

1.(10 points) Let $G = (V, E)$ be a graph. A **2-matching** in $G$ is a subgraph of G consisting of a set of vertex disjoint cycles.

(a) Write down an integer linear program for finding a 2-matching that contains the maximum number of vertices.

(b) Does the LP relaxation of your LP always give the optimum integer solution?

(c) Write down the dual of the linear program in (b). Is this a strong dual for the integer problem? Can you give any graph theoretic interpretation of the dual?

2. (10 points). Consider four inequalities of the form $a_i x \leq b_i, i = 1, \ldots, 4$, where $a_i$ and $x$ are $n$-vectors, and $b$ is a scalar. Show how to formulate the constraint: at least two of the inequalities must be satisfied.

3. (10 points) Let $A$ be an $m$ by $n$ matrix of nonnegative integers. Consider the following problems:

P1: we want to choose a maximum weight subset of elements of $A$ such that no two elements are from the same **row** of $A$.

P2: we want to choose a maximum weight subset of elements of $A$ such that no two elements are from the same **row** and **column** of $A$.

For each problem, decide if it can be solved by the greedy algorithm: choose elements from $A$ in decreasing order by weight, rejecting any element if it violates the given condition. Justify your conclusions by showing that the underlying problems either exhibit a matroid structure, or do not.