Integer Programming 308-761B
Class Test March 1, 2001

1.(10 points) Give a weak dual to the problem of finding a maximum cardinality matching in a graph. State both the original problem and the dual problem as integer linear programs. State (without proving) conditions for which the dual is a strong dual.

2. (5 points) Show how to efficiently model the following conditions in an integer linear program.

(i) $x = 2, 4,$ *or* 6.

(i) $x = 2, 4,$ *or* 5.

3. (5 points) Consider the following statement:

A matrix A of full row rank is totally unimodular if and only if all the vertices of the polyhedron

$$P = \{x : Ax \le 1\}$$

are integer valued.

Either prove the statement or give a counterexample and suggest a corrected statement.

4. (10 pts) Let E be a finite set, and I be a family of independent subsets of E. In class we said the family is a matroid if it satisfies property (1):

(1): For every subset A of E, if S and T are maximal independent subsets of A then they have the same cardinality.

Consider property (2):

(2): Let I_1 and I_2 be independent sets with $|I_1| = |I_2| + 1$. Then there is some element e of $I_1 - I_2$ such that $I_2 \cup \{e\}$ is an independent set.

Prove that (1) implies (2) and vice versa.