

First Class Test - COMP566

October 11, 2007

8:35 - 9:55am

No books or notes

Please state precisely any theorem that you use.

1 (5 marks). In a cow farm each cow receives a daily minimum of 100 units of calcium, 20,000 calories, and 1,500 units of protein. To accomplish this there are two types of feed available. Each kilogram of Cow Chow Feed costs \$1.50 and supplies 40 units of calcium, 1600 calories, and 900 units of protein. In contrast, each kilogram of the Moo Town Buffet Feed costs \$2.00 and supplies 80 units of calcium, 1000 calories, and 800 units of protein. Develop (but do not solve) an LP to get the cheapest mix of feeds to satisfy the cows' dietary requirements. Also write down the dual of this problem.

2.(15 marks). It is has been claimed that $x_1 = 2$, $x_2 = 3$, $x_3 = 0$ is an optimum solution to the following LP:

$$\max 8x_1 + 3x_2 - x_3$$

$$3x_1 + x_2 + x_3 \leq 9$$

$$5x_1 + 2x_2 - 2x_3 \leq 16$$

$$x_3 \leq 4$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Give three **different** proofs of the claim using any **three** of the following four methods.

- (a) Exhibit the final dictionary.
- (b) Run the final iteration of the revised simplex method.
- (c) Use the duality theorem.
- (d) Use the complementary slackness theorem.

Be sure to give a careful justification of your work.

3.(10 marks) (a) Consider an LP problem in standard form: $\max cx, Ax \leq b, x \geq 0$. Show that it is impossible that there are two dictionaries D and D' for this system with the following properties.

- (i) In D there is a basic variable such that in its row all coefficients are non-positive and the constant (ie. the b value in this row) is negative.
- (ii) In D' there is a cobasic variable such that it has positive coefficient in the objective row, and all the coefficients in its column are **strictly** positive.

(b) Can a similar claim be made if (ii) is replaced by (ii')? Give a proof or counterexample.

- (ii') In D' there is a cobasic variable such that it has positive coefficient in the objective row, and all the coefficients in its column are **non-negative**.