Second Class Test - COMP566A

November 17, 2005 8:35 - 9:55am

Each of the questions have equal weight. No books or notes

1(a) State the properties of a cutting plane for an integer linear programming problem.

(b) Consider the Knapsack Problem,

$$\max c^T x, \quad a^T x \leq b, \quad x_j \in \{0, 1\}, \; j = 1, \ldots, n.$$

where c, a are vectors of length n consisting of positive integers.
Let $S \subseteq \{1, 2, \ldots, n\}$ be such that

$$\sum_{i \subseteq S} a_i > b.$$

Prove that

$$\sum_{i \subseteq S} x_i \leq |S| - 1$$

is a cutting plane for this problem.

(c) Consider the knapsack problem with $c = (4, 5, 6)$, $a = (3, 4, 5)$ and $b = 10$. Find the solution of the LP relaxation. Find a cutting plane of the type mentioned in (b), and show that it satisfies the properties you described in (a).

2. Suppose you have an implementation of the Network Simplex Method available. Show how you could **efficiently** use it to solve the following problem. You are given a directed network with a specified source vertex s and sink vertex t. Each edge is assigned a maximum capacity, and also a cost. Your job is to find a maximum flow in the network from s to t that minimizes the total cost.

3. (a) State the conditions for a Nash Equilibrium (NE) for a two person game, with players Alice and Bob, defined by matrices A and B.
(b) Consider the game:

$$A = \begin{bmatrix} 0 & 4 & 5 \\ 4 & 0 & 5 \\ 3 & 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 & 6 \\ 0 & 4 & 3 \\ 5 & 5 & 3 \end{bmatrix}$$

Suppose Alice plays the pure strategy (0,0,1)and Bob plays (3/4, 1/4, 0). Show this is a NE.
(c) With Alice playing (0,0,1), find the whole range of strategies that Bob can play, each of which forms a NE.