First Class Test - COMP566A

October 13, 2005 8:35 - 9:55am

Each of the three questions have equal weight.

No books or notes

Please state precisely any theorem that you use.

1(a) A shephard can keep her sheep on public land. In any year, the maximum number of sheep she can keep is 100. Suppose she keeps N sheep in a given year. Due to overuse of the land, if $N \geq 50$, then the next year she may keep at most 200-2*N sheep. After 3 years, the land will be turned into a shopping centre. The shephard wants to maximize the total number of sheep she can keep over the three year period. Formulate this as a linear programming problem.

(b) Guess an optimal solution for this problem, and verify it by duality.
Hint: It seems likely that in year 3 the shepherd should graze 100 sheep.

2. Consider the following linear programming problem, where the slack variables are already included.

$$\begin{aligned} \textit{maximize} \quad & z = 2x_1 + 3x_2 + x_3 \\ \textit{subject to} \quad & x_1 + 2x_2 + 3x_3 + x_4 = 6 \\ & 2x_1 + x_2 + 2x_3 + x_5 = 4 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0. \end{aligned}$$

The problem was solved by the dictionary method but unfortunately only a part of the final dictionary remains.

$$?? = ?????????????????????? - 2/3x_4 + 1/3x_5$$

$$?? = ???????????????????????? + 1/3x_4 - 2/3x_5$$

$$z = ???$$

Without going through the simplex method, figure out the complete final dictionary. Does this dictionary in fact give an optimum solution? If so prove it using the duality theorem or complementary slackness theorem. If not explain why not and give a better solution.

3(a) By considering the dual problem or otherwise, show that the problem

$$\begin{aligned} \textit{maximize} \quad & 8x_1 + 3x_2 + 4x_3 \\ \textit{subject to} \quad & 3x_1 + x_2 + -2x_3 \le 5 \\ & 4x_1 - x_2 + x_3 \le 4 \\ & x_1,\ x_2,\ x_3 \ge 0. \end{aligned}$$

has no bounded solution.

(b) Suppose we add the constraint $x_1 + x_2 + x_3 \le 1$ so that the problem becomes bounded. Verify that the solution $x_1 = 1, x_2 = 0, x_3 = 0$ is optimal using duality.