Second Class Test - COMP566A

November 11, 2004 1:05 - 2:25pm

1. (a) Consider the integer linear program: $\max c^T x,\ Ax \leq b,\ x \geq 0$, x integer, where all data b, c, A are integers. Describe how to derive a cutting plane from the objective function.

(b) Suppose that the c vector is not an integer vector, but contains fractions. Show by a small example (two variables are enough) that the procedure described in (a) is invalid, and may cut off the optimum solution.

2. (a) State the *minimax theorem* for a two player zero sum game based on an m by n matrix A. Define the *value* of the game.

(b) Define what is meant by the term *Nash Equilibrium* as applied to a two player game based on two m by n matrices A and B.

(c) Prove or Disprove the following statement:

> A two person zero sum game can have Nash Equilibrium strategies (x, y) such that the expected payoff to each player is **not** equal to the value of the game.

3. Consider the linear program: $\min c^T x, Ax = b, x \geq 0$ and its dual $\max b^T y,\ A^T y + s = c, s \geq 0$.

(a) Write down the complementary slackness conditions. Derive a set of equations to define a generic Newton step in direction $(\Delta(x), \Delta(y), \Delta(s))$, with centreing parameter τ.

(b) Prove for the Newton direction defined in part (a) that $\Delta(x)\Delta(s) = 0$.