First Class Test - COMP566A

October 12, 2004 1:05 - 2:25pm

1. A cargo airplane has three compartments: front, centre and rear. Their weight capacities are, respectively, 12, 18 and 10 metric tons. Their space capacities are respectively 70, 90 and 50 cubic meters. Three types of cargo are available for transport: 20 tons of cargo A, 26 tons of cargo B and 25 tons of cargo C. The space requirements per ton of cargo are respectively: 5, 7 and 6 cubic meters. The profit for each ton of cargo transported is respectively 320, 400 and 290 dollars. **To maintain balance of the airplane, the weight of cargo in each compartment must be in the same proportion as the weight capacities of the compartments.** Formulate an LP to decide how many tons (fractions allowed) of each cargo should be transported to maximize profit.

2. Consider the following resource-allocation problem and the accompanying optimal dictionary:

$$\begin{array}{rll} \textit{maximize} & 4x_1 + 6x_2 + 5x_3 & \\ \textit{subject to} & 3x_1 + x_2 + 2x_3 \leq 64 & \textit{(Resource 1)} \\ & x_1 + x_2 + x_3 \leq 20 & \textit{(Resource 2)} \\ & x_1 + 2x_2 + 3x_3 \leq 30 & \textit{(Resource 3)} \\ & x_1,\ x_2,\ x_3 \geq 0. & \end{array}$$

$$\begin{aligned} x_1 &= 10 + x_3 - 2x_5 + x_6 \\ x_2 &= 10 - 2x_3 + x_5 - x_6 \\ x_4 &= 24 - 3x_3 + 5x_5 - 2x_6 \\ z &= 100 - 3x_3 - 2x_5 - 2x_6 \end{aligned}$$

(a) Prove optimality using either the Duality Theorem or the Complementary Slackness Conditions.

(b) Determine the range on the coefficient of x_1 in the objective function for which the current basis remains optimal.

(c) A new product is proposed. It is estimated that each unit of this new product consumes 4 units of resource 1, 2 units of resource 2 and 1 unit of resource 3. What is the minimum unit profit (ie. coefficient in the objective row) for this new product to be profitable to manufacture?

3.(a) Suppose you have an implementation of the revised simplex method available, and an LP problem with 1000 variables and 100,000 constraints. Would you recommend solving the problem as is? If so why, and if not how would you recommend solving the problem?

(b) Suppose you have an LP in the form $\min c^T x,\ \ Ax \le b,\ \ x \ge 0$. The b vector contains both positive and negative components, but the c vector is non-negative. Explain how to solve this problem using the simplex method **without using Phase I.**