Discrete Optimization - 308-566A

FIRST CLASS TEST No books, notes, calculators or cheating!

October 10, 2002 1:05 - 2:20pm

(5 pts) 1. Formulate the following LP and write down its dual. Try to find some interpretation of the dual variables. You do not need to solve the LP.

You have 10,000 dollars to invest in some combination of four investments. The annual yields are 3%, 5%, 7% and 15%. Each investment has a risk rating between 1(low) and 10(high). The ratings are respectively 1, 3, 5, 10. You want to maximize your annual yield, but will tolerate an average risk per dollar invested of at most 4.

(15 pts) 2. Consider the following LP.

$$\begin{aligned} \textit{minimize} \quad & z = x_2 + x_3 \\ \textit{subject to} \quad & x_1 - x_2 \le -5 \\ & x_3 \le 4 \\ & x_2 - x_3 \le 8 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

(a) Construct the dictionary corresponding to the basic feasible solution $x_1 = 0$, $x_2 = 5$, $x_3 = 4$.

(b) If the dictionary is not optimal, pivot until you get to an optimal solution.

(c) Give the optimal dual solution, and verify the complementary slackness condition.

Hint: Change the minimization to maximization!

(5 pts) 3. Consider any LP max $z = c^T x$, $Ax \le b$, $x \ge 0$, that has been converted to a starting dictionary. Prove that it is impossible to pivot this dictionary (by any number of pivots) to dictionaries D_1 and D_2 such that

(a) In D_1 there is a cobasic variable for which the signs of all coefficients in its column in D_1 are strictly positive.

(b) In D_2 there is an equation which reads

$$x_i = -b'_i - \sum_{j \in N} a'_{ij} x_j$$

where $b'_i > 0$ and $a'_{ij} > 0$, $j \in N$. N is the set of cobasic indices for D_2.