Discrete Optimization - 308-566A

FIRST CLASS TEST

October 4, 2001 1:00 - 2:30pm

(5 pts) 1. Write down the dual of the following LP:

$$\begin{aligned} \textit{maximize} \quad & 2x_1 \quad + 2x_4 \\ \textit{subject to} \quad & x_1 + 2x_2 + 3x_3 \le 4 \\ & x_1 \quad - 2x_3 + 3x_4 \ge 8 \\ & -3x_2 + x_3 = 6 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

(15 pts) 2. Consider the LP

$$\begin{aligned} \textit{maximize} \quad & 3x_1 - 4x_2 + 2x_3 \\ \textit{subject to} \quad & -2x_1 \le 4 \\ & x_1 \quad - 2x_3 \le 8 \\ & -3x_2 + x_3 \le 6 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

(a) Use the **weak duality theorem** to prove that this problem is unbounded.

(b) Solve the problem by the dictionary method using Dantzig's (greatest coefficient) rule, and give the unbounded solution.

(5 pts) 3. Let A be an m by n matrix, and let x be an n-vector and b a m-vector. Show that the system:

$$Ax \le b, \quad x \ge 0 \qquad (*)$$

has no solution if and only if there is an m vector y such that

$$yA \ge 0, \quad by < 0, \quad y \ge 0$$

Hint: Consider the LP with objective function zero and constraints (*).