Linear Programming - 308-566A

CLASS TEST October 31, 2000 1:05 - 2:25pm

You may state and use the weak or strong duality theorems without proving them.

1(a)(2 marks) If a primal linear program is infeasible what are the possible outcomes for the solution of the dual linear program?

(b)(3 marks) If the system of inequalities $Ax \le b$ has no solution, what conclusions can you draw about the outcome of solving the linear program

$$\min by \quad A^T y = 0, \quad y \ge 0?$$

Justify your answer.

2. Consider the following dictionary that arises from a general linear programming problem with upper and lower bounds, max $z = cx, \ Ax = b \ , l \le x \le u$:

$$x_1 = 12 + x_4 - x_5 + x_6 - 2x_7$$

$$x_2 = -4 - x_4 + x_5 - x_6 + 2x_7$$

$$x_3 = 5 + x_4 \qquad + x_6 - 2x_7$$

$$\max\ z = \quad 5x_4 - 4x_5 + 3x_6 - 2x_7$$

$$2 \le x_j \le 5 \qquad j = 1, 2, \ldots 7$$

(a)(3 marks) Fix x_4, x_5, x_7 at their upper bounds, and x_6 at its lower bound. Compute the values the basic variables and z.

(b) (12 marks) Consider an iteration of the simplex method with upper and lower bounds, starting from this dictionary. For **each of the cobasic variables:**

(i) decide if it would result in an increase in the objective function if it were chosen in Step 1 of the simplex method, and if so

(ii) compute the new values of each variable and z.

3. Consider an LP of the form max cx, $Ax \le b$, $x \ge 0$. Supposing you solved it by hand using the dictionary method, and after terminating you found out that you had written down the problem incorrectly. In each case below, state with brief justification whether the solution you obtained is still valid, and if not, explain precisely what actions you would take to avoid re-solving the LP from the beginning.

(a) (5 marks) You got an unbounded solution. In the objective row, you mistakenly wrote wrote down $10x_1$ instead of $-10x_1$, and x_1 is basic in the final dictionary you found.

(b) (5 marks) You got an optimum solution. In the first constraint you mistakenly wrote down $b_1 = 10$ instead of $b_1 = -10$, and x_1 is basic in the final dictionary you found.