308-566 Discrete Optimization
Homework 4 Due: Tuesday, November 5 2002, 5pm
MC308

You may use maple or matlab to solve systems of equations.

1. Consider the following LP that you can think of as a production problem with 4 products and 3 resources:

$$\begin{aligned} \textit{maximize} \quad & 15x_1 + 8x_2 + 10x_3 + 12x_4 \\ \textit{subject to} \quad & x_1 + 2x_2 + x_4 \le 20 \\ & x_1 + x_2 + x_3 + x_4 \le 54 \\ & 2x_1 + x_3 + x_4 \le 36 \\ & x_1, x_2, x_3, x_4 \ge 0. \end{aligned}$$

(a) It is claimed that the solution $x_2 = 10$, $x_3 = 36$ is optimum. Verify this claim by computing the final dictionary directly from this information.

(b) Suppose that a fourth resource constraint $3x_1 + 2x_2 + 2x_3 + x_4 \le 80$ is added to the problem. Reoptimize using the dual simplex method.

(c) Suppose a new product with cost coefficient 16 is now added, that requires respectively 1,2,1,1 units of the four resources. Reoptimize from your solution to (b) using the revised primal simplex method.

2. Chvatal, Exercise 19.1

3. Prove the following statement by giving an algorithm and proving it is correct (see Chvatal P. 296). Use the result to show that the vertex/edge adjacency matrix A with rows ordered $v_2, v_3, ..., v_n$ is upper triangular.
Let T be a tree on n vertices and let v_1 be any vertex. Show the remaining vertices can be labelled $v_2, v_3, \ldots, v_n$ such that for every $i \ge 2$ there is precisely one edge $v_i\ v_j$, where $1 \le j \le i - 1$.

This labelling can be thought of as a way of drawing the tree starting at vertex v_1, adding an edge at a time while maintaining connectivity.