308-566 Discrete Optimization - I
Homework 1 Due: Thurs September 26, 2002

1. For any LP problem, we can construct a directed graph on the feasible bases, showing which pivots are permissible under the simplex method. The vertices of the graph are labelled by the bases B_k, $k = 1, 2, \ldots$. An edge joins B_i to B_j if there is a valid simplex pivot between the dictionary with basis B_i to the dictionary with basis B_j. Construct this graph for Komei Fukuda's example below. Show which edges in the graph would be followed by Bland's smallest index rule.

$$\max x_1 - 2x_2 + x_3$$

$$2x_1 - x_2 + x_3 \leq 0$$

$$3x_1 + x_2 + x_3 \leq 0$$

$$-5x_1 + 3x_2 - 2x_3 \leq 0$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

2. Construct an example of a linear program with three variables and the following property. At first the value of z increases, then it stays constant for at least one pivot, then it increases again.

3. General Motors produces three types of cars:gas guzzler, compact and sub-compact, getting respectively 15, 25 and 30 miles per gallon (sorry, about the non-metric units, it is a US firm.) The profits per car sold are are respectively 700, 560 and 200 dollars. The maximum sales for each kind of car are respectively 400,000, 800,000 and 800,000 cars. Goverment regulations the average gas consumption over all cars sold must equal or exceed 26 miles per gallon. Formulate the profit maximization problem as an LP and solve it with the simplex method using Dantzig's rule. You should give the initial dictionary, a list of pivots, and the final dictionary.

You may find that using Maple or a similar package is useful for checking your arithmetic.

Policy on Late Assignments: -10% per day, including weekends. Assignments are due in class. Hand in late assignments to Bohdan Kaluzny, MC232.