CS566 Fall 2008 - Assignment 1 Solutions
(Note: your solutions should be more thorough, these solutions are just a sketch)

Exercise 1

(a) Let x_1 and x_2 be the number of units shipped from warehouse A to retail outlet 1 and 2 respectively and let x_3 and x_4 be the number of units shipped from warehouse B to retail outlet 1 and 2 respectively. The natural formulation of the problem is:

$$\begin{aligned} \min \quad & 6x_1 + 8x_2 + 4x_3 + 3x_4 \\ s.t. \quad & x_1 + x_2 \leq 68 \\ & x_3 + x_4 \leq 80 \\ & x_1 + x_3 = 36 \\ & x_2 + x_4 = 72 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Converting this to standard form we get:

$$\begin{aligned} \max \quad & -6x_1 - 8x_2 - 4x_3 - 3x_4 \\ s.t. \quad & x_1 + x_2 \leq 68 \\ & x_3 + x_4 \leq 80 \\ & -x_1 - x_3 \leq -36 \\ & -x_2 - x_4 \leq -72 \\ & x_1 + x_3 \leq 36 \\ & x_2 + x_4 \leq 72 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Solving this with lp_solve or CPLEX, the solution obtained is: $x_1 = 28$, $x_2 = 0$, $x_3 = 8$ and $x_4 = 72$ with an optimal cost of \$416.

Exercise 1 (b)

To obtain the balance we can add constraints:

$$\begin{aligned} & x_1 + x_2 \leq 1.1x_3 + 1.1x_4 \\ & x_3 + x_4 \leq 1.1x_1 + 1.1x_2 \end{aligned}$$

to the LP. The question did not state if a unit could be subdivided or not, some people choose to solve the LP assuming the units could not be subdivided (ie. an integer program) the solution obtained with this assumption is: $x_1 = 36$, $x_2 = 16$, $x_3 = 0$ and $x_4 = 56$ with an optimal cost of \$512. Assuming that the units can be broken up into pieces the soluion is: $x_1 = 36$,$x_2 = 15.4286$,$x_3 = 0$ and $x_4 = 56.5714$ with an optimal cost of \$509.143. Either approach was considered correct.

Exercise 2

The first step is to add slack variables to the LP to obtain the initial dictionary.

$$
\begin{aligned}
z &= x_1 + 3x_2 - 2x_3 \\
x_4 &= 6 + 2x_1 - x_2 - 3x_3 \\
x_5 &= 10 - x_1 - x_2 - 4x_3 \\
x_6 &= 6 - x_2 - 2x_3 \\
x_7 &= 34 + 3x_1 - 5x_2 - 14x_3
\end{aligned}
$$

So the initial basis is $\{x_4, x_5, x_6, x_7\}$. By the smallest subscript rule x_1 is the variable with a positive coefficient in the objective row and the smallest subscript, so x_1 will enter the basis. By Bland's rule, x_5 will leave the basis. This results in dictionary 2:

$$
\begin{aligned}
z &= 10 + 2x_2 - 6x_3 - x_5 \\
x_1 &= 10 - x_2 - 4x_3 - x_5 \\
x_4 &= 26 - 3x_2 - 11x_3 - 2x_5 \\
x_6 &= 6 - x_2 - 2x_3 \\
x_7 &= 64 - 8x_2 - 26x_3 - 3x_5
\end{aligned}
$$

By the smallest subscript rule x_2 enters the basis and x_6 leaves by Bland's rule. The resulting dictionary is:

$$
\begin{aligned}
z &= 22 - 10x_3 - x_5 - 2x_6 \\
x_1 &= 4 - 2x_3 - x_5 + x_6 \\
x_2 &= 6 - 2x_3 - x_6 \\
x_4 &= 8 - 5x_3 - 2x_5 + 3x_6 \\
x_7 &= 16 - 10x_3 - 3x_5 + 8x_6
\end{aligned}
$$

Since all the coefficents in the objective row are negative in the last dictionary, the current solution is optimal and has an objective value of 22 where $x_1 = 4$, $x_2 = 6$ and $x_3 = 0$.

I've included a sample of how you can use maple to solve the system of equations and obtain your dictionaries.

```
> consts := {z = x1 +3*x2-2*x3, x4 = 6 +2*x1 - x2 -3*x3, x5 = 10 - x1 - x2 -4*\
> x3, x6 = 6 - x2 -2*x3, x7 = 34 + 3*x1 -5*x2-14*x3};
consts := {z = x1 + 3 x2 - 2 x3, x4 = 6 + 2 x1 - x2 - 3 x3,

    x5 = 10 - x1 - x2 - 4 x3, x6 = 6 - x2 - 2 x3,

    x7 = 34 + 3 x1 - 5 x2 - 14 x3}

> solve(consts,{z,x7,x4,x1,x6});
{x6 = 6 - x2 - 2 x3, x1 = -x5 + 10 - x2 - 4 x3, z = -x5 + 10 + 2 x2 - 6 x3,
```

```
    x4 = 26 - 2 x5 - 3 x2 - 11 x3, x7 = 64 - 3 x5 - 8 x2 - 26 x3}

> solve(consts,{z,x7,x4,x2,x6});
{x2 = -x5 + 10 - x1 - 4 x3, x6 = -4 + x5 + x1 + 2 x3, x4 = -4 + 3 x1 + x5 + x3,

    x7 = -16 + 8 x1 + 5 x5 + 6 x3, z = -2 x1 - 3 x5 + 30 - 14 x3}
```

Exercise 3

(a) For a given LP,

$$\begin{array}{ll} max & \sum_{i=1}^{n} c_i x_i \\ s.t. & \sum_{i=1}^{n} a_(i,j) x_(i,j) \leq b_j \ \ j = 1, ..., m \\ & x_i \geq 0, i = 1, ..., n \end{array}$$

one way to determine if a given inequality $\sum_{i=1}^{n} a_{i,k} x_{i,k} \leq b_k$ is redundant is to remove the inequality from the LP and solve a new LP with the new feasible region in the direction orthogonal to the inequality removed.

$$\begin{array}{ll} max & \sum_{i=1}^{n} a_(i,k) x_i \\ s.t. & \sum_{i=1}^{n} a_(i,j) x_(i,j) \leq b_j \ \ j = 1, ..., k-1, k+1, ..., m \\ & x_i \geq 0, i = 1, ..., n \end{array}$$

If the objective value of this LP is $\leq b_k$ then constraint k is redundant, otherwise it is not. You can construct and solve such an LP for each inequaliy to get the list of redundant inequalities.

(b) Following the procedure outlined in (a), you should have obtained that the fourth constraint $-3x_1 + 5x_2 + 14x_3 \leq 34$ is redundant.