COMP566 Discrete Optimization - I
Homework 3 Due: Tuesday November 4, 2008, beginning of class

Page numbers refer to Linear Programming, V. Chvatal

1. Solve the transhipment problem given in Figure 19.1, P. 292-5 with the following modifications:

(a) $c^T = [15, -10, 30, 10, 22, 5, 8, 20, 12, 5, -10, 18, 10, -40]$, instead of the vector on P. 295. Show the optimum solution and dual variables. Start with the tree solution in Figure 19.4. List all pivots.

(b) Now reverse the arcs 14 and 63. Use the same c vector as in part (a), except now $c_{41} = -10$ and $c_{36} = -10$. Solve this problem, starting from any initial tree solution. List all pivots.

2. A network G is called *strongly connected* if there is a directed path between every pair of nodes. It is called *acyclic* if it does not contain a directed cycle. Prove the following:

(a) If G is strongly connected, the transhipment problem is feasible for any supply/demand vector b which sums to zero.

(b) If G is not strongly connected, there exists a supply/demand vector b which sums to zero, for which the transhipment problem is infesible.

(c) If G is acyclic, the transhipment problem is either infeasible or bounded, for any c vector.

(d) If G is not acyclic, there exists a c vector for which the transhipment problem is unbounded.

3.(a) Do exercise 1.8, P. 11, and solve using lp_solve or CPLEX. Give the optimum primal and dual solutions.

(b) Use sensitivity analysis and the dual simplex method to determine tight upper and lower bounds on the number of radios to be delivered, for which the solution in part (a) remains optimum.