COMP566 Discrete Optimization - I
Homework 2 Due: Tuesday October 7, 2008, beginning of class

By an LP in standard form, I mean max $z = c^T x, s.t.\ Ax \le b,\ x \ge 0$, where A is a m by n matrix.

1. (a) Solve this problem using the 2-Phase Simplex method, using Dantzig's rule (maximum positive cost coefficient, ties broken by minimum index).

$$\max 5x_1 + 3x_2 + 4x_3$$

$$2x_1 - x_2 + x_3 \le -10$$

$$3x_1 + 2x_2 - 2x_3 \le 30$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

List each the entering and leaving variables for each pivot and show the final dictionary after each phase.
(b) Same as part (a) but instead of $b_1 = -10$ set $b_1 = -20$.
(c) For your answers to part (a) and (b) give an appropriate certificate of correctness.

2. Consider an LP in standard form and its dual, with slack variables added in the usual way. Let x be a feasible primal solution and y be a feasible dual solution.
(a) Prove that

$$b^T y - c^T x = \sum_{j=1}^{n} x_j y_{m+j} + \sum_{i=1}^{m} x_{n+i} y_i$$

This is known as the duality gap.
(b) Use part (a) and the duality theorem to prove Theorem 5.2 (Complementary Slackness).

3. Consider an LP in standard form with optimal solution x^* and optimum objective value z^*. The optimum solution is *unique* if no other feasible solution obtains objective value z^*.
(a) Suppose that all basic variables in x^* are strictly positive. Give a necessary and sufficient condition for the optimum solution to be unique based on the coefficients of the final dictionary, and prove it is correct.
(b) Construct two examples with at least three variables and three constraints, one which has a unique optimum solution, and one that does not.