COMP566 Discrete Optimization I

Homework 4

Due: Tuesday November 20, 2007 in class.

Typo correction to 1(a) below, made Nov 17

1. Let A be an m by n integer matrix, c be an integer n-vector and b be an integer m-vector. We saw in class that for an LP

$$\max c^T x, \; Ax \leq b, \; x \geq 0$$

and any m-vector $y \geq 0$ the inequality

$$\lfloor y^T A \rfloor x \leq \lfloor y^T b \rfloor$$

is valid for all the feasible **integer** solutions. It is a cutting plane if it is violated by the optimal solution of the LP.
(a) Show that if the optimum objective value for the LP is fractional then the dual variables y give a cutting plane if $y^T A$ is an integer vector.
(b) Illustrate this on the telephone switchboard problem on pp 11-1 to 11-3 of the handout. Do this by solving the LP (using lp_solve or CPLEX) and getting the dual variables. Derive the cutting plane (3), p. 11-3. Incorporate this in your LP and see if you now get an integer solution.

2. Suppose A is a totally unimodular square matrix. Which of the following three matrices are totally unimodular (proof or counterexample):

$$A^T, \quad (A, A), \quad (A, A^T)?$$

(The notation (A,B) means just place the m by m matrices A and B side by side creating a new matrix of dimension $2m$ by m).

3. In the handout p. 11-16 Vasek states that the fifth cut, given in the middle of 11-15, is equivalent to

$$x_1 \leq 1.$$

Verify this statement. (Hint: you may use any of the other four cuts as stated, without proof.)