COMP566 Discrete Optimization I

Homework 2 Due: Tuesday, October 11 , 2005

Page numbers refer to Linear Programming, V. Chvatal

1. Consider any dictionary for an LP in standard form, with basic solution $x = (x_1, x_2, \cdots, x_{n+m})$. Let $\bar{c}_j$, $j = 1, 2, \ldots, n+m$ be the coefficients in the row for the objective function in the dictionary. Define $y_i = -\bar{c}_{n+i}$, $i = 1, 2, \ldots, m$ and $y_{m+j} = -\bar{c}_j$, $j = 1, \ldots, n$. Show the complementary slackness conditions hold:

$$x_j y_{m+j} = 0, \;\; j = 1, \ldots, n \qquad x_{n+i} y_i = 0, \;\; i = 1, \ldots, m.$$

Note that this is true whether or not x is primal feasible or y is dual feasible.

2. First formulate and solve problem 1.8 p. 11 using lp_solve or other software. Give the optimium primal and dual variables. Give an interpretation of each dual variable, and the complementary slackness conditions.

3. Solve question 3 of homework 1 using the revised simplex method. Directly verify that the complementary slackness conditions hold for the optimal solution.

4. In the assignment problem there are n students and n jobs, and the suitability of student i to job j is c_{ij}. Each student must be assigned a job, and each job a student. Write down the problem of finding an assignment maximizing the total suitablility as an integer linear program. Drop the integrality condition, and write down the dual linear program. What is an interpretation of the dual variables and complementary slackness conditions?