COMP566 Discrete Optimization I
Homework 2 Due: Tuesday, October 5, 2004

Page numbers refer to Linear Programming, V. Chvatal

1. Consider the following series of LPs for n=1,2,3,...

$$z_n^* = \min \sum_{j=1}^{n} x_j$$

$$\sum_{i=1}^{t-1} x_i + n x_t \geq 1 \qquad t = 1, 2, 3, \ldots, n$$

Prove that $z_n^* = 1 - (1 - 1/n)^n$. Hint: Solve the dual.

2. Consider any dictionary for an LP in standard form, with basic solution $x = (x_1, x_2, \cdots, x_{n+m})$. Let $\bar{c}_j$, $j = 1, 2, \ldots, n+m$ be the coefficients in the row for the objective function in the dictionary. Define $y_i = -\bar{c}_{n+i}$, $i = 1, 2, \ldots, m$ and $y_{m+j} = -\bar{c}_j$, $j = 1, \ldots, n$. Show the complementary slackness conditions hold:

$$x_j y_{m+j} = 0, \;\; j = 1, \ldots, n \qquad x_{n+i} y_i = 0, \;\; i = 1, \ldots, m.$$

Note that this is true whether or not x is feasible.

3. First formulate and solve problem 1.6 p. 10 using lp_solve or other software. Then do problem 5.6, p. 70.

4. Solve question 2 of homework 1 using the revised simplex method. Directly verify that the complementary slackness conditions hold for the optimal solution.