COMP566 Discrete Optimization - I
Homework 3 Due: Thursday October 30, 2003

Page numbers refer to Linear Programming, V. Chvatal

1. Solve this problem using the Revised Simplex method, using Dantzig's rule (maximum positive cost coefficient, ties broken by minimum index).

$$\max 5x_1 + 3x_2 + 4x_3$$

$$2x_1 + x_2 + x_3 \leq 20 \qquad (Resource\ 1)$$

$$3x_1 + x_2 + 2x_3 \leq 30 \qquad (Resource\ 2)$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Suppose a new product x_4 can be produced and it uses 3 units of resource 1 and 2 units of resource 2. What is the minimum profit (coefficient c_4) at which it becomes profitable to make this product?

2. Consider the LP problem:

$$\textit{minimize} \quad z = bw - cx$$

subject to

$$Ax \leq b$$

$$A^T w \geq c$$

$$x \geq 0 \qquad w \geq 0$$

where c and x are n-vectors, b and w are m-vectors, and A is an m by n matrix.
(a) Prove that this problem has and optimal value of zero or is infeasible.
(b) Give examples for each of these cases with $m \geq 2$ and $n \geq 2$.

3. The Dirty manufacturing company can make three products, subject to two resource constraints. Its profit maximization LP is:

$$\begin{aligned} \textit{maximize} \quad & 5x_1 + 6x_2 + 9x_3 \\ \textit{subject to} \quad & x_1 + 2x_2 + 3x_3 \leq 50 \\ & x_1 + x_2 + 2x_3 \leq 30 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

A previous consultant found the optimum production schedule, given by the final dictionary:

$$x_1 = 10 - x_3 + x_4 - 2x_5$$

$$x_2 = 20 - x_3 - x_4 + x_5$$

$$z = 170 - 2x_3 - x_4 - 4x_5.$$

Each unit of production causes 1 ton of pollution. The government has put in a new environmental restriction which limits total pollution to 25 tons, ie. a new constraint:

$$x_1 + x_2 + x_3 \leq 25$$

(a) Find an optimal solution to the new problem by using the dual simplex method and the previous optimum dictionary. Do not solve the problem from scratch! (Use either dictionary method or revised dual simplex method)

(b) The Clean manufacturing company does not use up all of its pollution allowance, and will sell the right to pollute to the Dirty company. What is the maximum price per ton that Dirty will pay Clean for this right?