Directed graphs

- A **directed graph (digraph)** is a pair (V, E) where V is a finite set of vertices and E the set of *directed* edges.
- Each edge in E is an ordered pair of vertices, denoted (u, v) (thats an edge from u to v).
- A **directed path** is a sequence of vertices $\langle v_1, v_2, \ldots, v_k \rangle$ such that $(v_i, v_{i+1}) \in E$ for all $i = 1, 2, \ldots, k-1$.
- A **cycle** is a directed path that starts and finishes with the same vertex and visits any vertex at most once.
- A vertex u is **reachable** from v if there is a directed path from u to v.
- A **sink** is a vertex with no edges going out (NB different from Cormen). A **source** is a vertex with no edges going in.

Directed acyclic graph

A **directed acyclic graph (DAG)** is a directed graph with no cycles.

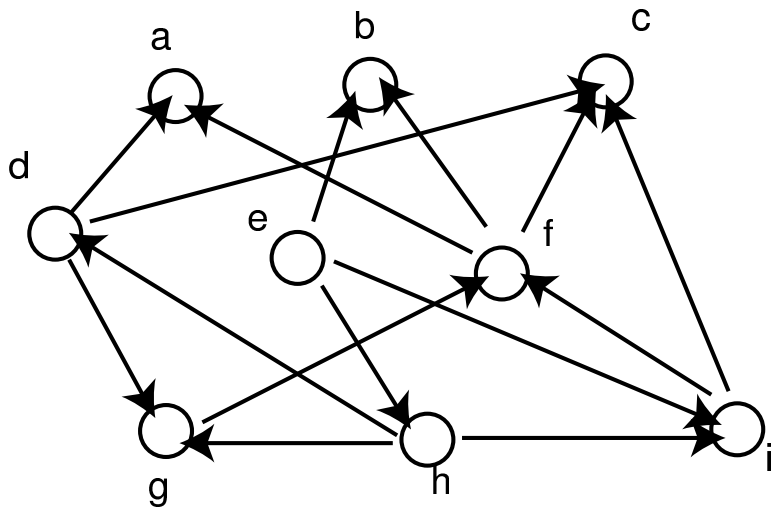
Lemma
Every DAG has a source (vertex with no incoming edges).

Proof
Choose a vertex a_0. If a_0 is a source, we are done. Otherwise there is an edge (a_1, a_0). Now consider a_1. If a_1 is not a source then there is an edge (a_2, a_1). Continue this process, noting that each time we examine a different vertex. Since the number of vertices is finite, we must eventually find a source.

Topological sort

A **topological sort** of a digraph is an ordering $v_1, v_2, \ldots, v_n$ of the vertex such that if (v_i, v_j) is an edge then $i < j$.

We can use induction and the previous lemma to prove:

Lemma
Every DAG has a topological sort

Example: