Algorithm Design Techniques - COMP 360

October 20, 2009 MIDTERM EXAM 11:35-12:50pm

No laptops, calculators, cell phones, books, notes or cheating.

1. (10 pts) (a) Define the term *stable matching.*

(b) Find **two** different stable matchings for the following preference lists. The men are labelled 1,2,3,4 and the women are labelled a, b, c, d.

1: c d a b 2: b d a c 3: a c b d 4: b a c d

a: 1 2 3 4 b: 1 3 2 4 c: 2 4 3 1 d: 3 1 2 4

(c) Consider a stable matching problem with n men and n women. Let M^1 and M^2 be two different stable matchings for these people. Suppose each woman who has a different partner in M^1 and M^2 chooses her preferred partner from these two men. Women with the same partner in both matchings do not change partners. Prove that this results in a matching, and that this matching is stable.

2. (10 pts) Recall that a directed grid graph is defined as follows. The nodes are pairs of integers (i, j), and edges are directed from (i, j) to $(i+1, j)$ and $(i, j+1)$. For each edge you are given an integer weight which may be positive, negative or zero.

(a) Write down a dynamic programming recursion to find the **maximum** weight path from (0,0) to (n,n), where $n > 0$ is a given integer.

(b) Suppose this code has already been implemented. How can you use it to find the **minimum** weight path from (0,0) to (n,n). Give a short proof that your method is correct.

3. (10 pts) (a) Define the terms flow, cut, source and sink. What is meant by a feasible flow? State the Max-flow Min-cut theorem.

(b) Starting with the flow given, solve the following max flow problem from source s to sink t, and find the minimum cut. You **must** show the residual graph for each step, and how to find the minimum cut from the final residual graph.