Question 3:

(a) Flow:
A function **f** that maps each edge **e** to a nonnegative real number. The value **f**(**e**) intuitively represents the amount of flow carried by edge e.

Cut:
A partition of the vertices of the graph into two parts, **A** and **B**, where the source **s** is in **A** and the sink **t** is in **B**.

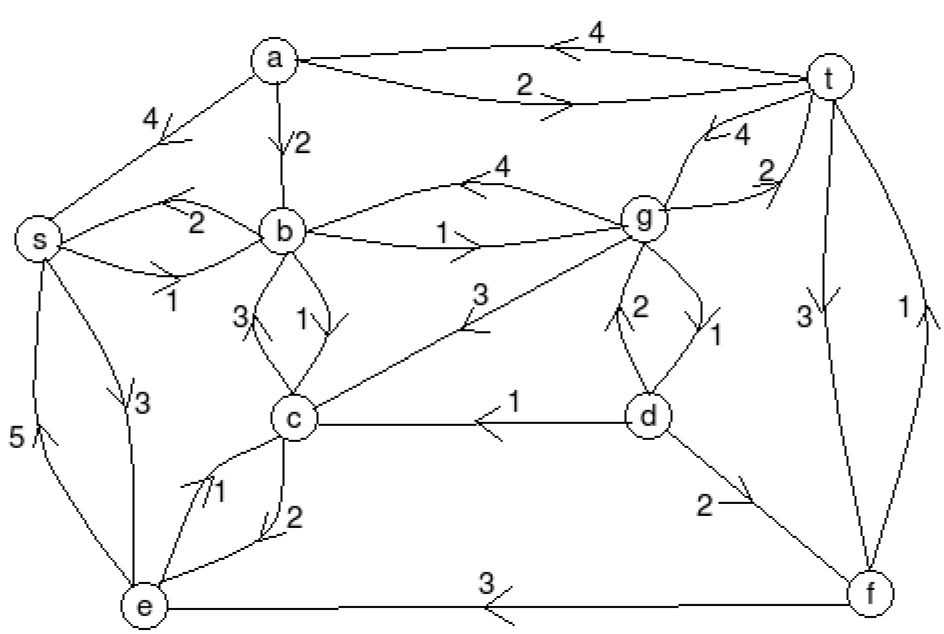
Source:
A vertex that generates flow.

Sink:
A vertex that absorbs flow.

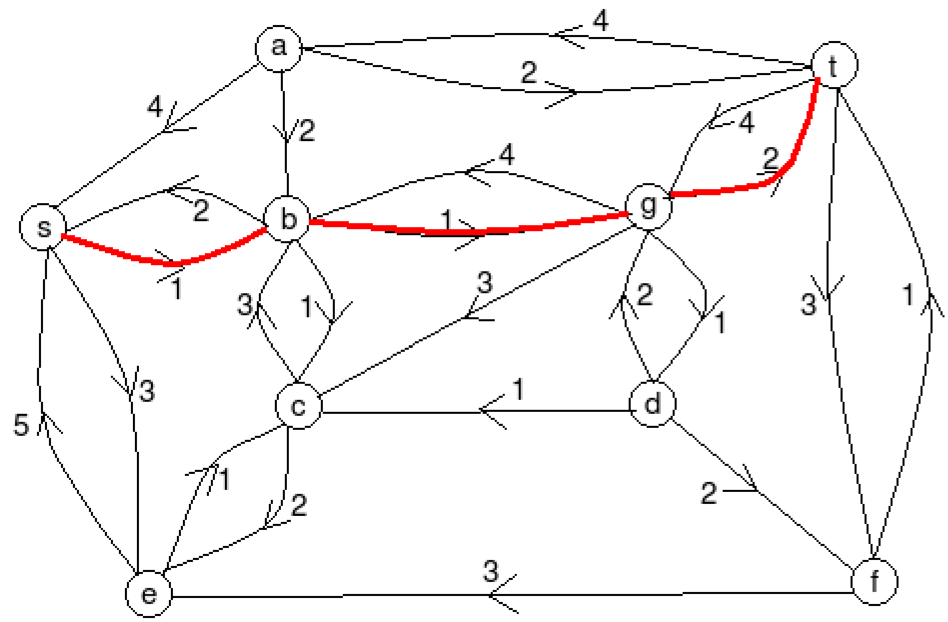
Feasible Flow:
A flow **f** such that it satisfies the following two properties: (capacity conditions) for each **e** in **E**, we have $0 \le f(e) \le c_e$, and (conservation conditions) for each node **v** other than **s** and **t**, we have $\sum_{e \text{ into } v} f(e) = \sum_{e \text{ out of } v} f(e)$.

The Max-Flow Min-Cut Theorem:
In every flow network, the maximum value of an s-t flow is equal to the minimum capacity of an s-t cut.

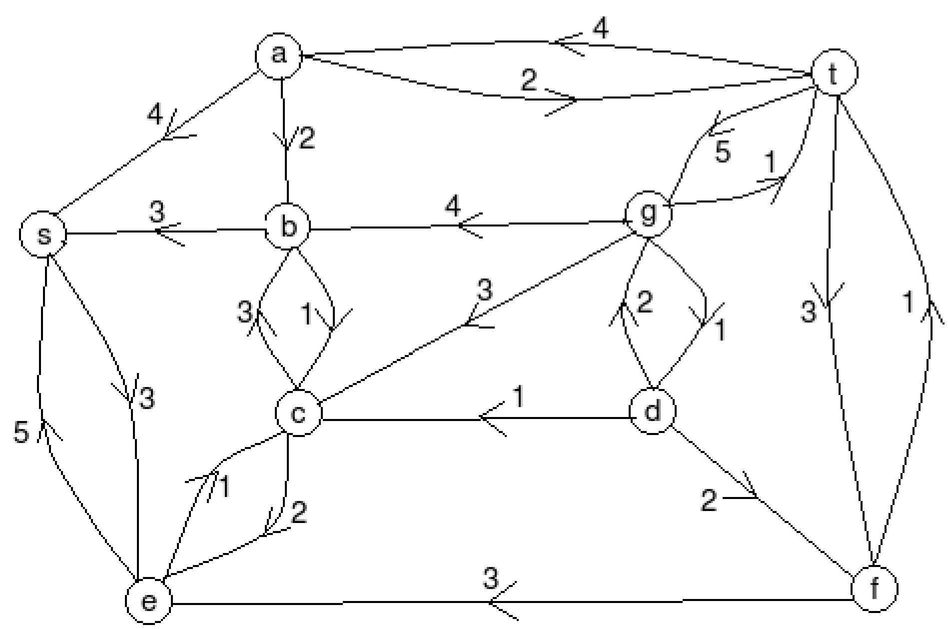
(b) **Yellow/Cream Exam**
The initial residual graph:



The path s, b, g, t in the above residual graph:



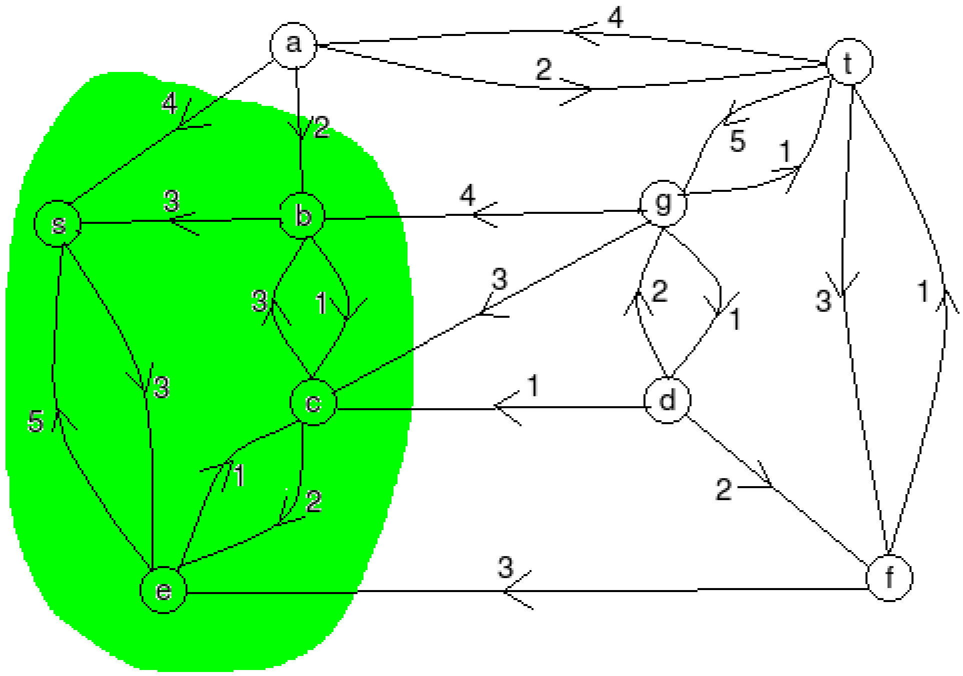
We can send 1 more unit of flow through this path. Once we do we get this new residual graph:



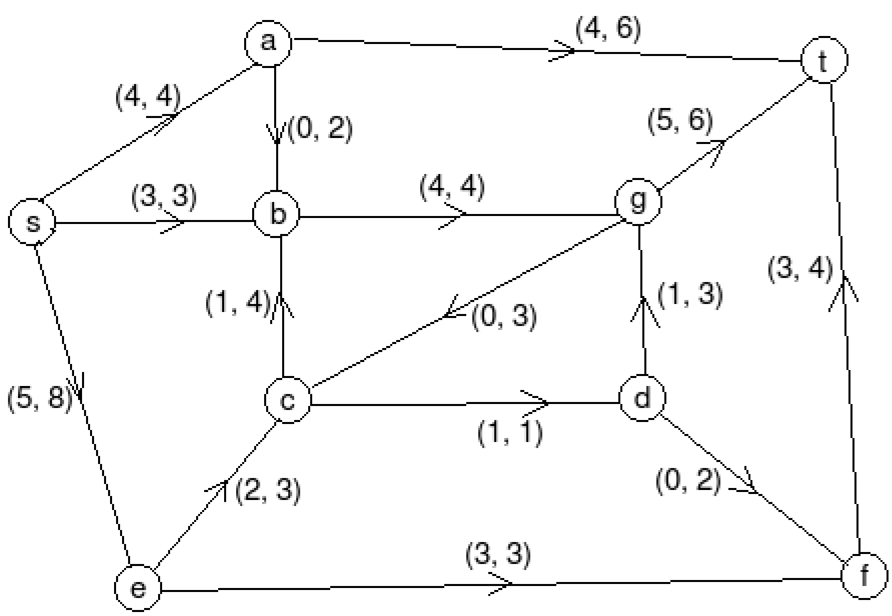
There are no more s-t paths in the residual graph, so we stop.

We can find the minimum s-t cut by doing a depth-first search on the vertices of the residual graph starting from the source s. All nodes found are in set A and all nodes not found are in set

B. The minimum cut is A = {s, b, c, e} and B = {a, d, f, g, t}, as illustrated here:

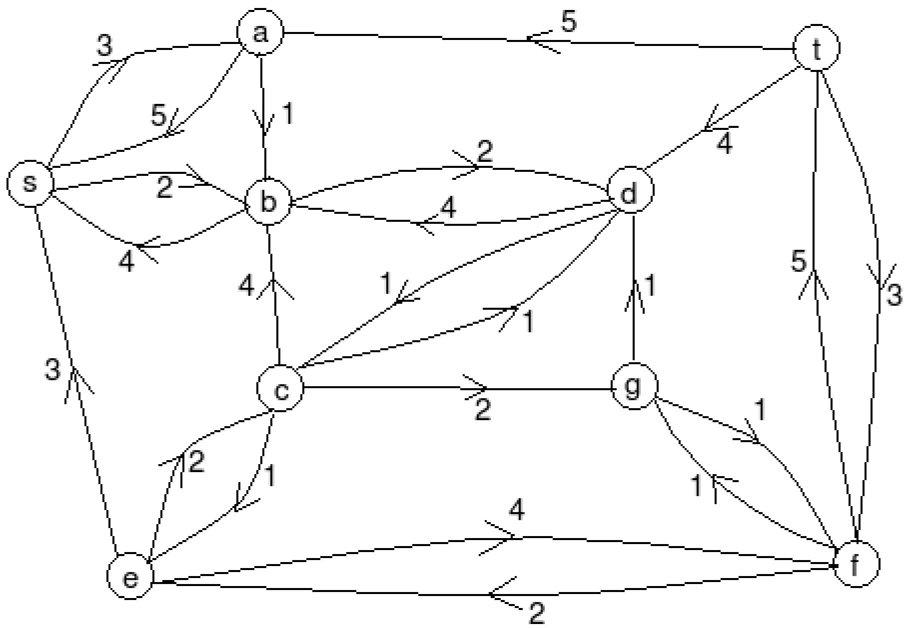


The max flow has value 12:

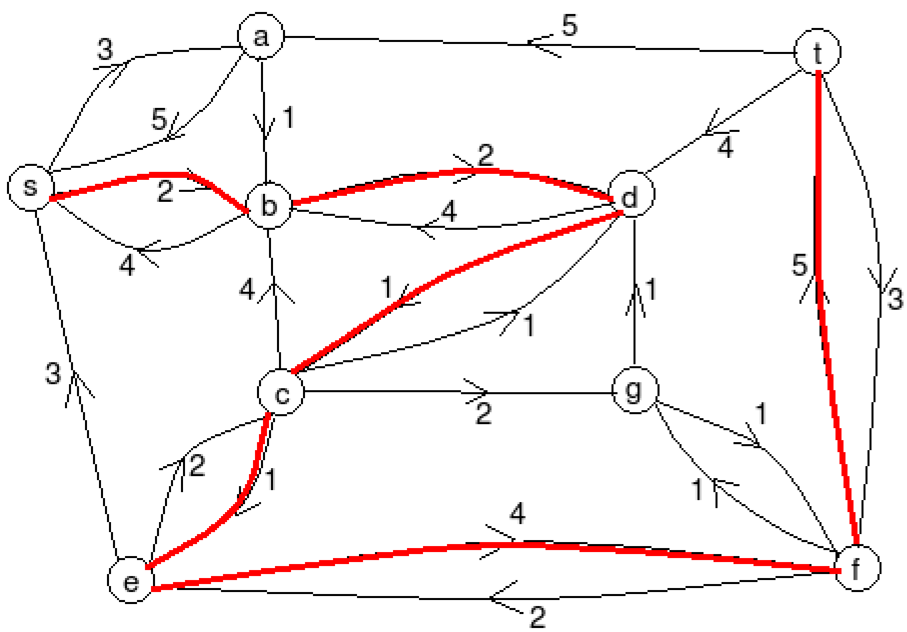


White Exam:

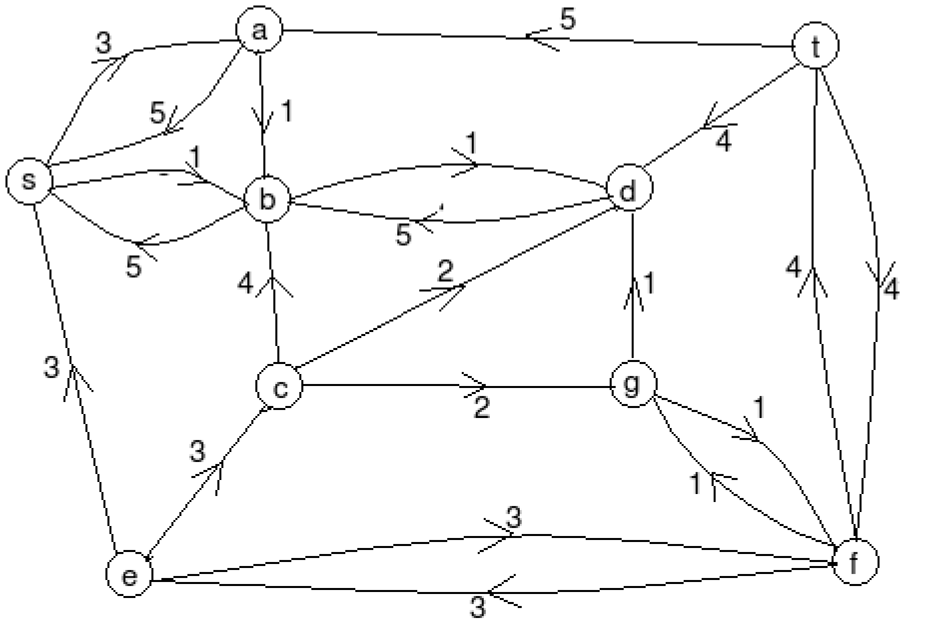
The initial residual graph:



The path s, b, g, t in the above residual graph:

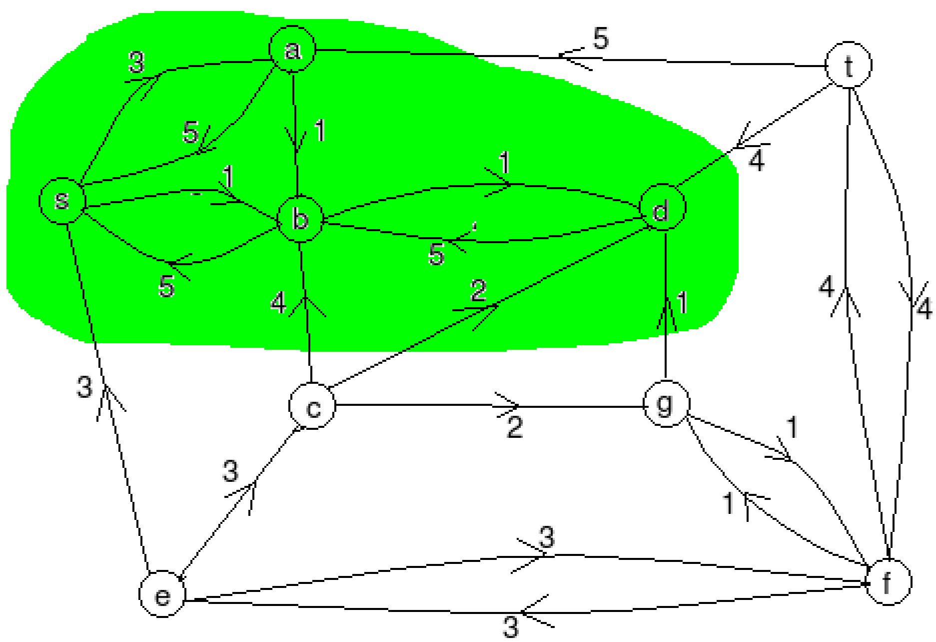


(c) We can send 1 more unit of flow through this path. Once we do we get this new residual graph:



There are no more s-t paths in the residual graph, so we stop.

We can find the minimum s-t cut by doing a depth-first search on the vertices of the residual graph starting from the source s. All nodes found are in set A and all nodes not found are in set B. The minimum cut is A = {s, a, b, d} and B = {c, e, f, g, t}, as illustrated here:



The max flow has value 13: