STUDENT NAME:________________________________ STUDENT NUMBER:______________

Faculty of Science

FINAL EXAMINATION

COMPUTER SCIENCE COMP 360B

Algorithm Design Techniques

Examiner: Dr. D. Avis Wednesday, December 16, 2009
Associate Examiner: Dr. C. Weibel 2-5 pm

Instructions

Answer directly on this exam paper.

You may use the back of the page if necessary.

Calculators are not allowed

Additional paper to be supplied for rough work. This will not be graded.

No cell phones, books, notes or cheating.

This exam consists of 6 questions, and has 7 pages.

Translation dictionaries only are allowed

Grades: 1._______2._______3._______4._______5._______6._______TOTAL______/60

(10 pts) 1. (a) State three properties for a weighted selection problem to be solvable optimally by the greedy selection algorithm.

(b) You are given a bipartite graph and asked to find the matching with largest number of edges. Can this be solved by the greedy selection algorithm? Either show it satisfies the three properties in (a), or give an example to show how one of the properties fails.

(c) Suppose on a given day at Trudeau airport, n airplanes arrive and leave. It is suspected that illegal luggage is being loaded onto some of the planes. There is one inspection team, and it wants to observe the maximum number of planes. They can only observe one plane at a time, and they must observe it from its arrival time to its departure time. Also it takes 10 minutes to move the inspection team from one plane to another. Give a greedy algorithm that can be used to give an optimum solution to their problem.

(10 pts) 2. In the partition problem you are given a set of n integer weights $a_1, a_2, \ldots, a_n$, and are asked to partition them into two subsets each weighing exactly half the total sum of the weights, if such a partition is possible. Suppose you know in advance that the total of all the weights is less than or equal to n^2. Design a dynamic programming algorithm that runs in $O(n^3)$ time to solve this problem, and state it in pseudo code. Your algorithm should explicitly output the partition when one exists.

(b) Illustrate your algorithm on the 5 weights: 3, 2, 4, 4, 7

(10 pts) 3. Find a maximum $s - t$ flow and minimum $s - t$ cut in the network below, starting from the given flow. Each edge is labelled with a pair (flow, capacity). Your answer should contain:
(a) The residual graph for the given network.
(b) The augmenting flow.
(c) The residual graph for the updated network.
(d) The minimum capacity $s - t$ cut.

(10 pts) 4.(a) Prove that if all capacities in a network flow are integers then there is a maximum flow f for which every flow value $f(e)$ is an integer. (You may make use of the Ford-Fulkerson algorithm without proving it is correct.)

(b) Give an example to show that the original Ford-Fulkerson algorithm can run in exponential time as a function of the input size. Describe how to use scaling to modify the algorithm to run in polynomial time. (You need not prove the modified algorithm runs in polynomial time.)

(10 pts)5. Consider the following linear program in standard form:

$$\max z = 4x_1 - x_2 + x_3, \text{ subject to:}$$
$$x_1 - x_2 + x_3 \le 4$$
$$-2x_1 + x_2 - x_3 \le 6$$
$$x_1 + 2x_2 + x_3 \le 7$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

(a) Write down the dual linear program.

(b) State one method of verifying that a given solution to a linear program is in fact optimum.

(c) Verify that the solution $x_1 = 5$, $x_2 = 1$, and $x_3 = 0$ is an optimum solution to the problem above using the method you described in part (b). (Hint: $y_1 = 3, y_2 = 0, y_3 = 1$ is a feasible dual solution.)

(10 pts) 6. The 3-OF-4 SAT problem is stated as follows:
Input: n logical variables $x_1, x_2, \cdots, x_n$ and a set of m clauses each containing 4 terms (ie. x_i or $\bar{x}_i$) for 4 distinct variables x_i.
Question: Can you assign truth values to the variables so that **at least one and at most three** of the terms in each clause are true.
(a) Formulate the 3-OF-4 SAT problem as an integer linear program. Illustrate your formulation on the following 3-OF-4 SAT input: (here "+" is "or" and "." is "and")

$$(x_1 + \bar{x}_2 + \bar{x}_3 + x_5) \,.\, (x_2 + \bar{x}_3 + \bar{x}_4 + x_6) \,.\, (\bar{x}_1 + x_2 + x_3 + \bar{x}_5) \,.\, (x_2 + x_3 + x_4 + \bar{x}_6)$$

(b) Show that if a 3-OF-4 SAT instance has a "Yes" answer then there are always at least **two distinct** truth assignments to the variables that give a "Yes" answer. Also give an example of a 3-SAT instance that has **only one** truth assignment satisfying all the clauses.