Algorithm Design Techniques - COMP 360

February 14, 2008 MIDTERM EXAM 10:05-11:25

No laptops, calculators, cell phones, books, notes or cheating.

(10 pts) 1. (a) For a group of 100 students you have for each student s a list of the students s knows, and a list of the students s does not know. You want to make a party with the largest number of students such that everybody at the party:
(i) knows at least two other people at the party, and
(ii) does not know at least two other people at the party.
Solve this using the Greedy method, and give a proof of correctness.
Hint: If all 100 students satisfy (i) and (ii), the party will have 100 people. Otherwise some student ....

(b) State the optimum cache maintennance problem, and give a greedy algorithm to solve it. Illustrate on the sequence
a,b,c,d,a,d,e,a,d,b,c
where the cache has length 3 and is initially loaded with a,b,c. Give another optimum solution that is **not** obtained by the greedy algorithm.

(10 pts) 2. (a) State the weighted interval scheduling problem, and give a dynamic programming recursion to solve it that runs in **polynomial time.**
(b) Show how to modify your solution to accept an additional parameter k, and to find a set of **exactly** k intervals with maximum total weight satisfying the conditions of the interval scheduling problem. If no such set exists, your algorithm should report this.

(10 pts) 3. It is required to schedule m lifeguards to work at a swimming pool over a period of n days. Each lifeguard i supplies a list S_i of the days when she is willing to work. There are d lifeguards required each day, and no lifeguard is allowed to work more than t of the n possible days.
(a) Show how to formulate this problem as a maximum flow problem.
(b) Suppose no such schedule is possible, how will you be able to recognize this from the output?
(c) Solve the problem for the given data:

$$m = 4,\ n = 6,\ d = 2,\ t = 3,$$

$$S_1 = \{1, 2, 3, 5\}, S_2 = \{2, 3, 4, 6\}, S_3 = \{1, 2, 4, 5, 6\}, S_4 = \{3, 5, 6\}.$$

Draw our the network showing the capacities and the maximum flow. Write down the feasible schedule.