Faculty of Science

FINAL EXAMINATION

COMPUTER SCIENCE COMP 360

Algorithm Design Techniques

Friday April 18, 2008 9am - 12 noon

Examiner: Prof. David Avis

Associate Examiner: Prof. Patrick Hayden

Instructions

Calculators are not allowed

No cell phones, books, or notes.

This exam consists of 6 questions, and has 3 pages including this cover page.

Dictionaries are not allowed

Answer in EXAM BOOK

1. (10 pts)
McGill needs help assigning summer research assistants to research projects. A group of n students have been selected and there are n profs, each having one project. Each student ranks each project in order of preference, and each prof ranks each student.

(a) Define the term *stable matching.*

(b) Describe briefly a polynomial time algorithm that allows McGill to find an assignment of students to projects that is a stable matching.

(c) Suppose that among all stable matchings, McGill would like one for which each prof gets his highest ranked student possible. Explain how to do this efficiently.

(d) Illustrate your answer to 1(c) for the preference lists for projects a,b,c and students A,B,C given by

$$A: abc \quad B: acb \quad C: abc$$

$$a: BAC \quad b: BCA \quad c: CAB$$

This means, for example, that student A ranks project a first and project c last.

2. (10 pts) You are given an input string of n characters $x_1, x_2, \cdots, x_n$. It is required to decompose the string into words, of at least **three** letters each, that use consecutive characters of the string. So for example the string

C A P R I L L A T E R

decomposes into either

(i) A P R I L + L A T E R or (ii) C A P + I L L + A T E

You are given a function $dic(i, j)$ that returns "1" if $x_i, x_{i+1}, \ldots, x_j$ is a correct word, and "0" otherwise.
(a) Give an efficient algorithm that finds a decomposition using the maximum number of **characters** of the input string. (Eg. (i) above, which uses 10 of the 11 input characters.)
(a) Give an efficient algorithm that finds a decomposition using the maximum number of **words.** (Eg. (ii) above, which has 3 words, the maximum possible.)

3. (10 pts)
(a) Define what is meant by the Maximum Flow problem in a network. Be sure to define the terms *flow, capacity, source and sink.*
(b) Prove the following statement:

> If all edge capacities in the network are integers, then there is a maximum flow f for which on every edge e the flow value $f(e)$ is an integer. (You may state and use the Ford-Fulkerson algorithm without proving its correctness.)

(c) Give a small example of a network which has integer capacities but also has a maximum flow for which some flow values $f(e)$ are non-integer.
(d) Briefly show how to find a maximum matching in a bipartite graph using network flows. Show how the statement in (b) is crucial to the correctness of your algorithm.

4. (10 pts)
(a) Explain what is a linear program (LP) in standard form, and what is its dual.

(b) Describe the three possible outcomes of a linear programming problem. For each outcome, give a two dimensional example of an LP in standard form which has this outcome. Also give a two-dimensional sketch for each example, showing the constraints and illustrating the outcome.

(c) For each of the outcomes in 4(b), describe a certificate of correctness. Give such a certificate for each of your examples in 4(b).

5. (10 points)
(a) State the independent set (IND) and vertex cover (VC) problems. What is the relationship between these two problems.
(b) State the 3-SAT problem. Give a direct reduction of the 3-SAT problem to the vertex cover problem.
(c) Illustrate your reduction in part 5(c) by constructing the VC instance corresponding to the 3-SAT expression:

$$(x_1 + \bar{x}_2 + \bar{x}_3)(x_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_1 + x_2 + x_3)(x_2 + x_3 + x_4),$$

where "+" means "or".

6. (10 points)
DMA Snowcleaning removes snow from roofs that are about to collapse. There are two teams available to handle m jobs, and there is an estimated time $t_i, i = 1, \ldots, m$ required for each job. The company wants to assign the jobs to the teams in such a way as to minimize the time the last job is completed (after all, the snow may melt by itself eventually!) You are called in as a consultant.
(a) Explain why there is likely to be no efficient general algorithm for this problem, by relating it to an NP-complete problem you studied in the course.
(b) Give an integer programming formulation of the problem.
(c) Give an efficient algorithm for finding a solution which is no more than 3/2 longer than the optimum. You do not need to give the analysis of the heuristic.
(d) Suppose the times t_i are known to be integers between 1 and 8. Explain why you can now confidently claim to be able to solve the problem optimally, even for large values of m, by describing an $O(m^2)$ time algorithm.