Algorithm Design Techniques - COMP 360

October 18, 2007 MIDTERM EXAM 11:35 - 12:55

No calculators, cell phones, books, notes or cheating.

(10 pts) 1. Imagine a long straight road along which houses are spread out. House i is at distance d_i from the start of the road. You are asked to position the minimum number of cell phone antennae such that each house is at most 5 km from an antenna.
(a) Give a greedy algorithm to solve this problem.
(b) Show the **correctness** of your algorithm and give a **certificate of optimality** by developing the following idea: if there is a subset S of k houses such that each pair of houses in S is more than 10km apart, then at least k antennae are needed.

(10 pts) 2. You are given an input sequence of n positive integers $a_1, \cdots, a_n$ which sum up to the even number $2N$.
(a) Design a $O(nN)$ time algorithm to see if there is a subset of the numbers that sums to N. Is this algorithm polynomial in the input size.
(b) Illustrate your algorithm on the following input sequence with $n = 5, N = 8$:

$$5,\ 1,\ 4,\ 3,\ 3$$

(10 pts) 3. After a natural disaster there are n injured people (numbered $1, 2, \ldots, n$) each of whom must be taken by ambulance to a hospital. There are k hospitals, $H_i, i = 1, 2, \ldots, k,$ and hospital H_i can treat at most k_i people. For each hospital H_i there is a subset P_i of the injured people who have the correct medical insurance, and H_i will not accept any other people.
(a) Using network flows, design an algorithm for finding the maximum number of people that can be taken to a hospital that will treat them.
(b) Illustrate on the problem with

$$k = 3,\ n = 7,\ k_1 = k_3 = 2,\ k_2 = 3,$$

$$P_1 = \{1, 2, 6, 7\},\ P_2 = \{1, 2, 4, 7\},\ P_3 = \{3, 5\}.$$

Draw the network. Compute the maximum flow, showing the residual graph for the last two iterations. Show the minimum cut.